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HOW FRANKLIN'S ELECTRICAL MACHINE CAME TO THE FRANKLIN INSTITUTE *

BY

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One of the original Franklin electrical machines has long been a prized possession of The Franklin Institute. It is regularly on display in Franklin Hall. The archives of the Institute contain the letter of presentation that explains the circumstances under which this piece of equipment was given to the Institute in 1826.

The text of this letter[2] is:

To the Managers of The Franklin Institute.—

Gentlemen—

In requesting your acceptance of the accompanying Electrical Machine, I beg you to estimate its importance, not from any intrinsic value in itself, but from its having belonged to the venerable Philanthropist & Sage, whose name designates the Institution over which You preside.—It is believed to be the identical machine, with which he performed those Experiments that helped to immortalize him; and which are detailed in the philosophical Transactions of the period in which they were made.—

I have somewhere seen a statement, that Dr. Franklin's Electrical Machine was in the possession of General la Fayette, & graced an apartment at la Grange.—It is highly probable that the Doctor may have constructed a variety of Machines in the course of his long continued devotion in the pursuits of Electricity; and that some of them have found their way to distant parts.—The following statement, will however render it almost certain, that *this*, was his favorite Machine, & the longest preserved by its illustrious owner—

In the year 1792, a number of Medical Students of the University of Pennsa. associated to form a Chemical Society.—Amongst them was the late Dr. Wm. Bache, my particular friend & fellow Graduate; who began his Medical pursuits, under the

* Presented before the City History Society of Philadelphia, February 15, 1955.

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[2] The text of this letter was published in this JOURNAL, Vol. 251, p. 188 (1951).

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.

direction of the late Dr. Wistar, in the Year which terminated the mortal career of his immortal Grandsire.—Dr. Bache was the favorite Grandson of that Great Man, owning in part, to the congeniality of their pursuits; and the Machine in question, was a present to him a short time before his death.—It was highly esteemed by Dr. Bache, and was always regarded by him, as the one principally employed by Franklin.—The estimation in which he was held by the Chemical Society, soon raised him to a considerable standing in it; & he kindly loaned, and subsequently presented to the Society, which he trusted would be permanent, this memorial of his Grandfather's affection.—Dr. Woodhouse, Professor of Chemistry in the University, & then president of our Society, was permitted the use of this Machine, in the pursuit of his lectures; but whilst in his possession, the Society was broken up, and was never again organized.—At the death of Dr. Woodhouse in 1809, I was appointed to succeed him in the Chemical department of the University, and purchased all his apparatus, amongst which I found the Machine I now transmit to you.—I have had it in my possession nearly 17 years; and have concluded that I could not find a better, nor more permanent location for it, than in the Cabinet of an Institution so happily conducted as Yours.—The Name your Institution bears, must incite to emulation and perseverance, its Members; for it would be the extreme of incongruity, to have that name associated with indifference and neglect.

Wishing you Gentlemen, uninterrupted success in your great & praiseworthy pursuits; & hoping that the adage "Esto perpetua," may be applicable to your Institution, I beg leave to subscribe myself, with much respect,

Yr. very obedient Servt.
John Redman Coxe
Augt. 4th. 1826—

Receipt of this letter is recorded in the minutes of the meeting of the Board of Managers of The Franklin Institute on September 6, 1826 (1)[3] in the following words:

A Communication was received from Dr. J. Redman Coxe presenting an Electrical Machine which formerly belonged to the illustrious Franklin and with wish [which] he made his earliest experiments in relation to that fluid.

At the quarterly meeting of The Franklin Institute on October 16, 1826, the following action was taken:

Resolved, That the Institute have heard with particular pleasure, of the Donation by our fellow citizen, Dr. John Redman Coxe, of an Electrical Machine which formerly belonged to the immortal Franklin;—a donation which is the more valuable, as it may tend to cherish, by the presence of a sensible memorial, a vivid recollection, not only of the great Scientific Merits, but also of some few of the peculiar attainments of that great Philosopher.

Resolved, That General Stephen Duncan, Walter R. Johnson, and George W. Smith be appointed a Committee to wait on Dr. Coxe, with a copy of the foregoing Resolution. **(2, 3)**

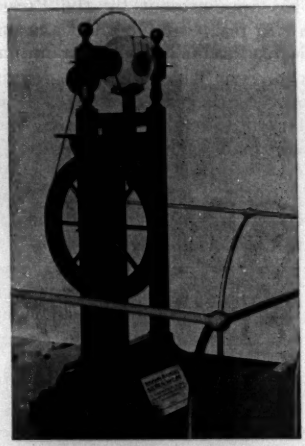
Coxe's letter not only enumerates the successive owners of the electrical machine, from Benjamin Franklin to The Franklin Institute, but also reveals that the machine was once the property of the Chemical Society of Philadelphia, and that that Society was "broken up" during the lifetime of Woodhouse. The Chemical Society to which Coxe refers

[3] The boldface numbers in parentheses refer to the references appended to this paper.

in his letter undoubtedly was the Chemical Society of Philadelphia[4] which will be discussed in the second article of this series.

WILLIAM BACHE

William Bache was the second child and second son of Richard Bache and his wife, Sarah Franklin, the daughter of Benjamin Franklin. He was born in Philadelphia, May 31, 1773, and died in Philadelphia,



Franklin's Electrical Machine

August 5, 1814 (6). This date of birth is also given in the Baptismal Register of Christ Church where he was baptized July 17, 1773; these dates of birth and baptism are entered in the folio lectern Bible which Benjamin Franklin gave to his daughter, Sarah, and which has been loaned to The Franklin Institute by her descendants, William Duane, Jr. and John P. Duane. The Interment Register of Christ Church records the burial of William Bache on August 7, 1814 in Christ Church Burial Ground.[5]

[4] This is evidenced by the year of origin (1792), the membership (medical students of the University of Pennsylvania), the fact that William Bache, in his dissertation (4), records himself as "Honorary Member and Annual President of the Philadelphia Chemical Society," and the fact that the Chemical Society met at the Laboratory on January 10, 1801, and elected officers, including Woodhouse as president, and Coxe as a vice-president and a lecturer (5).

[5] His grave is in Section A, and in the same row with those of his daughter, Emma Mary Bache, his father and mother, Richard and Sarah Bache, and his grandfather and grandmother, Benjamin and Deborah Franklin. Leach (8) gives the span of Bache's life as May 31, 1773 to August 5, 1814, and states that his death was the result of a drowning accident. The grave, now marked, must have been unmarked when the record of the burial ground inscriptions was published in 1864, since its inscription is not included in that compilation (9).

William Bache was graduated from the University of the State of Pennsylvania (A.B. 1790) and from the University of Pennsylvania (A.M. 1793, M.D. 1794) (7). His dissertation for the medical degree (4) was a study of the morbid effects of carbonic acid gas (carbon dioxide, fixed air) on healthy animals. It will be described in the third article in this series.

On July 2, 1802, Albert Gallatin, Secretary of the Treasury, notified William Bache of his appointment by the President of the United States as "Physician to reside at New Orleans under the act for the relief of sick and disabled seamen." The communication is addressed to Bache at Franklin, near Charlottesville, Virginia. The cited act (10) had been approved May 3, 1802. On June 2, 1802, Gallatin wrote to "Daniel Clarke," Consul of the United States at New Orleans, mentioning the appointment of Bache; "Daniel Clark," then in New York, replied on August 16, 1802. Both letters are concerned with the carrying into effect of the cited act. While Louisiana then was a French possession, it still had a de-facto Spanish governor; and Clark speaks of the "Spanish Government."

The papers mentioned in the preceding paragraph are the property of Sarah Madeline Hodge, the great-granddaughter of William Bache. She also possesses three certificates of appointment of William Bache, each signed by Thomas Jefferson, President of the United States, and by James Madison, Secretary of State. The first, dated April 2, 1804 is an appointment to be Surveyor for the Port of Philadelphia "during the pleasure of the President of the United States for the time being, and until the end of the next session of the Senate of the United States, and no longer." The second, dated November 22, 1804, is as Surveyor for the Port of Philadelphia "during the pleasure of the President of the United States for the time being." The third, also dated November 22, 1804, appoints Bache, Inspector of Revenue for the Port of Philadelphia, with the same tenure as the second.

Beginning in 1805 and ending in 1814, William Bache is listed as Surveyor of the Port of Philadelphia in various Philadelphia directories (James Robinson 1805–1811, Jane Aitken 1811, John A. Paxton 1813, B & T Kite 1814).

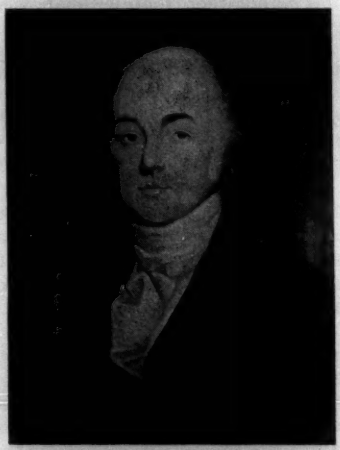
A prolonged search for a portrait or other likeness of William Bache has been fruitless.

JAMES WOODHOUSE

James Woodhouse, the founder and president of the Chemical Society of Philadelphia and custodian for it of the electrical machine, was a native of Philadelphia, the son of William and Ann Woodhouse, born November 17, 1770 and baptized August 22, 1771, according to the Baptismal Register of the United Churches (Christ Church and St. Peter's Church). He died in Philadelphia, June 4, 1809 (11, 12).

His funeral was held June 6 with interment in the Church Yard of St. Peter's Church. His tomb in Section B is marked by a monument (13).

Woodhouse was a graduate of both the University of the State of Pennsylvania (A.B. 1787, A.M. 1790) and the University of Pennsylvania (M.D. 1792) (14). His doctoral dissertation was devoted to the biochemistry of the persimmon (15). It will be discussed in the third article of this series. Woodhouse was elected "professor of chymistry" in the University of Pennsylvania on July 7, 1795 (16) when Joseph Priestley declined that chair to which he had been elected in the preceding November. Woodhouse also served as Dean of the Faculty of



JAMES WOODHOUSE

Portrait by Charles Willson Peale in the School of Medicine, University of Pennsylvania

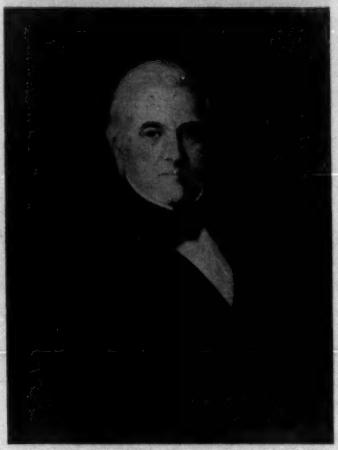
Medicine in the University; his tenure of that office in 1802 is mentioned by Edgar F. Smith, and in 1808 and 1809 by Robinson (17). In a comprehensive biography (18), Provost Smith pays tribute to Woodhouse as a medical officer in the United States Army, a physician in the yellow fever epidemic of 1793, an educator, author, editor, and research worker in the fields of laboratory experimentation, combustion, respiration, composition and decomposition of water, plant chemistry, chemical analysis, industrial chemistry, and the isolation of metallic potassium by ignition of either pearl ash (potassium carbonate) or potash (potassium hydroxide) with powdered soot.

Smith mentions the high regard in which Woodhouse was held by his students. This regard existed beyond the University. Thus Isaac

Abrahams, the author of a brochure on "The Powers of Chymistry, in Relation to Things Visible & Invisible; or a Walk through the Laboratory of the Skies" published at Philadelphia in 1801, dedicated his treatise to the Faculty of the University of Pennsylvania, but addressed his preface to Dr. Woodhouse, President of the Chymical Society, Philadelphia. Chapters are addressed to the Professor of Chymistry in the University.[6]

JOHN REDMAN COXE

John Redman Coxe, author of the presentation letter, was an original member of The Franklin Institute and became a life member in 1829. He and his grandfather, John Redman, were members of the Chemical Society of Philadelphia. Coxe was born in Trenton, New



JOHN REDMAN COXE

Portrait by Louise Wood in the School of Medicine, University of Pennsylvania

Jersey, on September 16, 1773, and died in Philadelphia on March 22, 1864. He graduated with the degree of Doctor of Medicine from the University of Pennsylvania in 1794. His thesis (19) was on inflammation; its biochemical portion will be presented in the third article in this series.

Coxe was a Trustee of the University of Pennsylvania from 1806 to 1809; its professor of chemistry from July 10, 1809 to July 7, 1818, and its professor of materia medica from July 7, 1818 to February 3, 1835 (20). The title of the professorship was expanded to include

[6] A copy of Abrahams' brochure is in the library of the American Philosophical Society.

pharmacy, for Coxe is designated as the present "incumbent of the Chair of Materia Medica and Pharmacy" in the Trustees' minutes in 1835 (20). This addition of pharmacy to the professorship apparently occurred in 1819 since the minutes of the medical faculty contain the notation on August 3 of that year: "Pharmacy added to the chair of Materia Medica" (21).

During his earlier years on the medical faculty Coxe, as its youngest member, acted as its secretary (21). Under the statutes of the University (22) "the medical professors shall, each in rotation, act as dean for one year, and it shall be the duty of the dean to arrange and conduct the business of examining the candidates for medical degrees." Under this statute Coxe functioned as dean in 1818 (20). However, the minutes of the medical faculty for March 20, 1819 (21) record: "the difficulties attending the Deanship, in consequence of the continual change of the Dean, suggested the propriety of making it permanent, in one person.....Dr. Coxe accepted the appointment—with the proviso, after engaging in it for 2 or 3 years, to decline its continuance, if he should be inclined so to do." Coxe was succeeded as dean by W. E. Horner in the summer of 1822 (21).

The activities of Coxe in the field of pharmaceutical education led to the establishment of the Philadelphia College of Pharmacy. On March 7, 1820, the Trustees of the University of Pennsylvania received a letter from Coxe suggesting the propriety of "sanctioning under the authority of the Board, by Licenses granted, after due examination, Apothecaries in their particular avocations" (23). The subject was referred to a committee which rendered a report and submitted resolutions on January 2, 1821 (24). This report was considered and the resolutions agreed to on February 6, 1821 (23). These resolutions instituted the degree of Master of Pharmacy, requested the Faculty of Medicine to report to the Board a proper form of Diploma and "a list of such Apothecaries in the city and liberties of Philadelphia as are desirous and in their opinion deserving of obtaining the degree of Master of Pharmacy, and unless sufficient reason to the contrary shall appear the degree of Master of Pharmacy shall be conferred on such individuals respectively." The resolutions also specified the requirements for this degree in pharmacy: A regular apprenticeship of at least three years with a respectable apothecary or Master of Pharmacy, a certificate of qualification from the Faculty of Medicine signed by its Dean and granted on the attestation of the professors of chemistry, materia medica and pharmacy after examination of the candidate, and a certificate of good moral character. In the future, in order to obtain the degree, the candidate was required to attend at least two courses of lectures in chemistry, materia medica and pharmacy in the University.

The form of diploma proposed for Master of Pharmacy was submitted to the Trustees on March 6, 1821. On April 3, 1821, the

Trustees issued a mandamus for a commencement on April 5 to confer the degrees of doctor of medicine and master of pharmacy. At that commencement 66 degrees in medicine were conferred; and the degree of Master of Pharmacy was conferred on 16 gentlemen, all of Philadelphia. The University conferred the Master of Pharmacy degree thereafter on but one candidate, on April 4, 1823 (23).

The medical faculty received an application for this degree in 1824 (21) but refused the request "as an examination previously is indispensible." However, George B. Wood, writing in 1827, stated: "The degrees conferred in the medical department are those of Doctor of Medicine and Master of Pharmacy," and enumerated the requirements for each degree (25).

The apothecaries of Philadelphia objected to the above plan of the University for pharmaceutical education, since it would place regulation of the profession of pharmacy in the hands of the University and its medical faculty (26). Accordingly they met in Carpenters' Hall on February 23, and March 13, 1821, and organized the Philadelphia College of Apothecaries which was incorporated as the Philadelphia College of Pharmacy by special act of the General Assembly of Pennsylvania approved March 30, 1822 (27). The change to its present corporate title, Philadelphia College of Pharmacy and Science, was by decree of Court of Common Pleas No. 1 of Philadelphia on April 26, 1920 (28).

On March 16, 1824, John Redman Coxe and Nathaniel Chapman (also a member of both the Chemical Society of Philadelphia and The Franklin Institute) were nominated; and, on April 20, 1824, they were duly elected Honorary Members of the Philadelphia College of Pharmacy (29).

This is the letter and these were the men that made it possible to own and display one of the Institute's most valued instruments—Benjamin Franklin's original electrical machine.

Coxe's letter kindled anew in the writer an interest in the Chemical Society of Philadelphia acquired during a conference in 1913 with Edgar Fahs Smith, then the Provost of The University of Pennsylvania. It stimulated study of several related subjects: the successive owners of the electrical machine, the relationship of the members of the Chemical Society of Philadelphia to the University of Pennsylvania and to The Franklin Institute, the early history of these institutions, the pioneer work in biochemistry at the University, and the origin of collegiate instruction in pharmacy in America.

Much hitherto unpublished information was found, chiefly in the manuscript records of local institutions. It is presented in the three following articles of this series:

The Chemical Society of Philadelphia and Certain Local Institutions
Pioneer Biochemical Researches at the University of Pennsylvania
The Academy Bell: The Franklin Institute and Old Philadelphia

Acknowledgments

Use has been made of the Edgar F. Smith Collection in the History of Chemistry, and of the following libraries in Philadelphia: American Philosophical Society, Church Historical Society, College of Physicians, Diocese of Pennsylvania, Franklin Institute, Genealogical Society of Pennsylvania, Historical Society of Pennsylvania, Library Company, and University of Pennsylvania. The author is indebted to William W. Montgomery for data from the manuscript records of Christ Church, and to the following persons for permission to consult written records in their custody: John E. Kramer, Registrar, Philadelphia College of Pharmacy and Science; Frederick J. Fee, then Assistant to the Secretary of the Board of Directors of City Trusts; and, at the University of Pennsylvania, John C. Hetherston, Secretary, Dr. Leonidas Dodson, Archivist, and Maud M. Tracy, Alumni Records Office. Sarah Madeline Hodge has supplied information and loaned documents concerning William Bache. Mary Stanley Field (Mrs. Mark Harvey) Liddell, the grand-daughter of Robert Maskell Patterson, has furnished information and suggested sources thereof concerning the Pattersons. The Rev. Dr. Alexander Mackie has permitted the author to glean data from the manuscript of his book, "Facile Princeps: The Story of Life Insurance in America," now in press. Illustrations have been furnished by Mr. Hetherston, Lea & Febiger, the Insurance Company of North America and the American Philosophical Society.

Manuscripts cited in the bibliography and/or the text include: charter books, commission books, church registers, minutes of corporations and other organizations, and papers in archives.

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(15) JAMES WOODHOUSE, "An Inaugural Dissertation on the Chemical and Medical Properties of the Persimmon Tree and the Analysis of Astringent Vegetables," Philadelphia, 1792, 34 pages.
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(17) JAMES ROBINSON, *Philadelphia Directory* for 1808, pages xlii–xliii; for 1809, pages xxxix–xl.
(18) EDGAR F. SMITH, "James Woodhouse: A Pioneer in Chemistry," Philadelphia, 1918, 299 pages.
(19) JOHN REDMAN COXE, "An Inaugural Essay on Inflammation," Philadelphia, 1794, 57 pages.
(20) Minutes of the Trustees of the University of Pennsylvania: Vol. 5, p. 305 (1806), pp. 351, 352 (1809); Vol. 6, pp. 252, 263 (1818); Vol. 8, pp. 140–150 (1835).
(21) Minutes of the Medical Faculty of the University of Pennsylvania, Vol. 2, p. 1 (1811), p. 61 (1819), p. 72 (1822), p. 104 (1824).
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THE MAGNETOSTATIC FORCE ON TWO CIRCULAR CYLINDRICAL CONDUCTORS CARRYING UNIFORM STEADY CURRENTS

BY

E. E. JONES [1]

ABSTRACT

The magnetic field due to the flow of steady currents in two parallel cylindrical conductors of circular sections set in a non-uniform external magnetic field is determined, the materials forming the conductors being magnetic in character. General expressions for the mechanical forces and the torques acting on the cylinders are deduced, and investigated for particular examples.

I. INTRODUCTION

The investigation of the magnetostatic field surrounding current-carrying conductors is of importance in view of the occasions in practice when such conductors are set in close proximity, and the forces exerted on them may be sufficiently large to cause instability or rupture when the currents flowing are large, for example under short-circuit conditions (1).[2] It has been indicated by Higgins (2) that very little investigation has been carried out concerning magnetic fields in regions of differing permeabilities, although the solution to the problem of two circular cylindrical conductors formed from non-magnetic material is well known (3), and the mechanical forces exerted on the conductors can be determined either by means of the Biot-Savart law of force, or by the more artificial method of deducing first the mutual inductance of the two conductors (4). In this paper the exact form of the magnetic field throughout the whole of space is determined when two parallel conducting cylinders of different permeabilities, having circular sections of different radii, are set in a fluid medium of given permeability in which there exists a non-uniform magnetic field. The cylinders are assumed to carry unequal steady currents which are uniformly distributed over their right sections, the proximity effect being neglected. The problem considered is of interest as in practice current-carrying conductors usually come under the influence of stray magnetic fields, which influence both the form of the magnetic fields around the conductors and the forces which act on them. An excellent bibliography which encompasses allied problems of this nature has been compiled by Higgins (5).

Since this paper was completed the author has encountered a solution to the more restricted problem of the determination of the magnetic field when the currents in the conductors are respectively equal in magnitude but opposite in direction, and when the effect of the external

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

magnetic field is not considered (6). The exclusion of these restrictions in the present paper brings out certain detail which is not evident in (6), but as the ultimate purpose is to deduce the forces on the cylinders, only the main steps in the theory for the determination of the magnetic field will be recorded in order to avoid any repetition.

The cylinders are assumed to be external to each other, their plane right sections defining the z-plane, where $z = x + iy$, and having circular contours corresponding to $\xi = \xi_1 < 0$, and $\xi = \xi_2 > 0$, in the coaxal net given by the transformation

$$\zeta = \xi + i\eta = \log (z - a)/(z + a). \tag{1}$$

Here a is a real constant, hence $z = a$ and $z = -a$ are the limiting points P and Q, respectively, of the coaxal system of circles. If $z - a = r_1 \exp i\theta_1$, and $z + a = r_2 \exp i\theta_2$, then

$$\xi = \log (r_1/r_2), \quad \eta = \theta_1 - \theta_2,$$

and then the point z is said to have bipolar coordinates (ξ,η). The curves $\xi = \text{const.}$ are circles of radii $|a \operatorname{cosech} \xi|$, and centers $(-a \coth \xi, 0)$. The point P corresponds to $\xi = -\infty$, and the point Q to $\xi = +\infty$. It can be shown that $\xi < 0$ in the region $x > 0$, and $\xi > 0$ in the region $x < 0$, and that η is a multi-valued coordinate. The transformation (1) can be rendered single-valued by introducing a cross-cut along the real axis in the region $-\infty < x < -a$, and in $a < x < +\infty$. It also follows that at infinity $\xi = 0$ and $\eta = 0$ when $y > 0$, whereas $\xi = 0$ and $\eta = 2\pi$ when $y < 0$.

The space inside the circle corresponding to $\xi = \xi_1$ will be referred to as region (1), and here $-\infty < \xi < \xi_1$; the space inside the circle $\xi = \xi_2$ will be referred to as region (2), and here $\xi_2 < \xi < \infty$; whereas at points external to both circles $\xi_1 < \xi < \xi_2$, and these constitute the region (3).

The magnetic materials forming the various media are assumed to be non-ferromagnetic, and in all the equations listed in this paper the rationalized mks system of units is employed (7, p. 16).

II. THE FIELD EQUATIONS

It is assumed that a current I_1 flows in region (1) with uniform density J_1, and a current I_2 flows in region (2) with uniform density J_2. With an obvious suffix notation

$$I_n = \pi a^2 J_n \operatorname{cosech}^2 \xi_n, \quad (n = 1, 2). \tag{2}$$

The magnetic vector potential in each region is directed parallel to the axes of the cylinders. If A_1, A_2, A_3 are the non-zero components of the magnetic vector potentials in the regions (1), (2), (3), respec-

tively, and μ_1, μ_2, μ_3 are the corresponding inductive capacities, then

$$\nabla^2 A_n = -\mu_n J_n, \quad (n = 1, 2, 3), \tag{3}$$

where it is assumed that $J_3 = 0$.

In all regions of space the divergence of the vector potential must be zero, and this follows immediately since the only non-zero component of the vector potential is that in the axial direction of the cylinders, and is a function of x and y only.

In terms of the bipolar coordinates (3) can be written in the form

$$\frac{\partial^2 A_n}{\partial \xi^2} + \frac{\partial^2 A_n}{\partial \eta^2} = -h^2 \mu_n J_n, \quad (n = 1, 2, 3), \tag{4}$$

where $h = a/(\cosh \xi - \cos \eta)$, and $J_3 = 0$.

On the surfaces separating two magnetic materials the boundary conditions demand that the vector potentials and the tangential components of the magnetic intensities are respectively continuous. The conditions are

$$A_n = A_3, \quad \mu_3 \frac{\partial A_n}{\partial \xi} = \mu_n \frac{\partial A_3}{\partial \xi}, \tag{5}$$

and must be satisfied when $\xi = \xi_n$, $(n = 1, 2)$, with $0 \leqslant \eta \leqslant 2\pi$.

At a large distance from both cylinders A_3 has a limiting value consisting of the sum of two parts. The first part A_0 is due to the magnetic field formed by the currents flowing in the conductors, and the second part A_0' is that due to the external magnetic field. At large distances the currents act as if they were parallel line-currents through the limiting points P and Q, hence

$$A_0 = -\frac{\mu_3}{2\pi}(I_1 \log r_1 + I_2 \log r_2).$$

But $r_1/r_2 \to 1$ as $r_1, r_2 \to +\infty$, hence the predominant part of A_0 at large distances is given by

$$A_0 = \frac{\mu_3}{4\pi}(I_1 + I_2) \log (\cosh \xi - \cos \eta), \tag{6}$$

where $r_1 r_2 = 2a/(\cosh \xi - \cos \eta)$.

In connection with this result it will be required to introduce a Fourier expansion of the form

$$\log (\cosh \xi - \cos \eta) = -\log 2 + |\xi| - 2 \sum_{n=1}^{\infty} \frac{1}{n} e^{-n|\xi|} \cos n\eta, \tag{7}$$

valid for $|\xi| < 0$, and $0 \leqslant \eta \leqslant 2\pi$.

Again for reasons which appear in detail in Section IV, it is assumed that for $\xi < 0$,

$$A_0' = \mu_3 \sum_{n=1}^{\infty} (b_n \cos n\eta + c_n \sin n\eta) e^{n\xi}, \tag{8}$$

and for $\xi > 0$ that

$$A_0' = \mu_3 \sum_{n=1}^{\infty} (f_n \cos n\eta + g_n \sin n\eta) e^{-n\xi}, \tag{9}$$

where b_n, c_n, f_n, g_n are known real constants.

III. SOLUTIONS OF THE FIELD EQUATIONS

By separating the variables it can be shown that a solution of the Laplace differential equation of the type given by (4), with $n = 3$, exists in the form

$$A = B + C\xi + F\eta + G\xi\eta + \sum_{n=1}^{\infty} (B_n \cos n\eta + C_n \sin n\eta) e^{n\xi} + \sum_{n=1}^{\infty} (F_n \cos n\eta + G_n \sin n\eta) e^{-n\xi}, \tag{10}$$

where the coefficients introduced are arbitrary real constants.

For solutions of the Poisson differential equation of the type given by (4), with $n = 1, 2$, a particular integral

$$- a^2\mu J \coth \xi/2(\cosh \xi - \cos \eta),$$

is added to (10). This extra term may be represented by a Fourier expansion

$$- a^2\mu J |\coth \xi| \{\tfrac{1}{2} + \sum_{n=1}^{\infty} e^{-n|\xi|} \cos n\eta\},$$

valid for $|\xi| < 0$, and $0 \leqslant \eta \leqslant 2\pi$.

Solutions of (4) can now be formed which are single-valued and finite at all points of the respective regions. Thus with a similar notation to that of (10),

$$\frac{A_1}{\mu_1} = B' + \tfrac{1}{2}a^2 J_1 \coth \xi + \sum_{n=1}^{\infty} \{(B_n' + a^2 J_1 \coth \xi) \cos n\eta + C_n' \sin n\eta\} e^{n\xi},$$

$$\frac{A_2}{\mu_2} = B'' - \tfrac{1}{2}a^2 J_2 \coth \xi + \sum_{n=1}^{\infty} \{(F_n' - a^2 J_2 \coth \xi) \cos n\eta + G_n' \sin n\eta\} e^{-n\xi},$$

$$\frac{A_3}{\mu_3} = B + \left\{C - \frac{I_1 + I_2}{4\pi}\right\} \xi + \sum_{n=1}^{\infty} (F_n \cos n\eta + G_n \sin n\eta) e^{-n\xi} + \sum_{n=1}^{\infty} \left\{\left(B_n + b_n - \frac{I_1 + I_2}{2\pi n}\right) \cos n\eta + (C_n + c_n) \sin n\eta\right\} e^{n\xi},$$

for $\xi < 0$, and

$$\frac{A_3}{\mu_3} = B + \left\{C + \frac{I_1 + I_2}{4\pi}\right\} \xi + \sum_{n=1}^{\infty} (B_n \cos n\eta + C_n \sin n\eta) e^{n\xi} + \sum_{n=1}^{\infty} \left\{\left(F_n + f_n - \frac{I_1 + I_2}{2\pi n}\right) \cos n\eta + (G_n + g_n) \sin n\eta\right\} e^{-n\xi}, \quad (11)$$

for $\xi > 0$.

It is noticed that terms involving powers of η have been ignored, thus rendering the expressions single-valued. Terms involving ξ and $e^{-n\xi}$ have been excluded from the expression for A_1 as these diverge at the point where $\xi \to -\infty$ in region (1), and similarly terms in ξ and $e^{n\xi}$ have been excluded from the expression for A_2 as these diverge at the point where $\xi \to +\infty$ in region (2). The magnetic vector potential A_3 also has the correct form at infinity where $\xi = 0$, $\eta = 0$ or 2π, providing that the remaining terms are convergent under these conditions.

IV. THE COMPLEX POTENTIAL FUNCTIONS

The vector potential component A_3 is a harmonic function, hence it can be assumed to be the imaginary part of a complex potential function w, which is a function of z only. In general the magnetic intensity $\mathbf{H}$ and the vector potential $\mathbf{A}$ are connected by the relation $\mu\mathbf{H} = \text{curl } \mathbf{A}$, therefore the components H_x, H_y of $\mathbf{H}$ in region (3) can be determined from the equation

$$\mu_3(H_x - iH_y) = \frac{dw}{dz}. \quad (12)$$

If $\xi < 0$, then from (11), the complex potential w can be written in the form

$$w = -\frac{i\mu_3 I_1 \zeta}{2\pi} + i\mu_3 \sum_{n=1}^{\infty} \left\{\left(\alpha_n - \frac{I_1 + I_2}{2\pi n}\right) e^{n\zeta} + \beta_n e^{-n\zeta}\right\}, \quad (13)$$

where

$$\alpha_n = B_n + b_n - i(C_n + c_n), \quad \beta_n = F_n + iG_n. \tag{14}$$

It was stated in Section II that at large distances from the cylinders the external magnetic field was associated with a vector potential A_0' given by (8) and (9). This potential may be interpreted as the imaginary part of a complex potential function w_0' in the form

$$w_0' = i\mu_3 \sum_{n=1}^{\infty} \gamma_n e^{n\zeta}, \tag{15}$$

where $\xi < 0$, and by definition $\gamma_n = b_n - ic_n$, and $\gamma_{n0} = b_n + ic_n$. It is well known that a general type of two-dimensional magnetic field can be represented in terms of a complex potential function

$$w' = \mu_3 \sum_{n=1}^{m} H_n z^n, \tag{16}$$

where m is a positive integer, and H_n is in general a complex constant. Application of (12) leads to the determination of the components of the magnetic intensity of the field. The coefficients H_n, and thus the magnetic intensity of the field, can be determined experimentally by means of a harmonic analysis of the radial component of the field or the gradient of the field as measured with a dipole or quadrupole search coil (8).

The inverse transformation of (1) is

$$z = -a \coth (\zeta/2), \tag{17}$$

hence in terms of ζ Eq. 16, with $\xi < 0$, can be replaced by

$$w' = \mu_3 \sum_{n=1}^{m} H_n a^n \sum_{r=0}^{\infty} a_{nr} e^{r\zeta}, \tag{18}$$

where

$$(1 + e^{\zeta})^n (1 - e^{\zeta})^{-n} = \sum_{r=0}^{\infty} a_{nr} e^{r\zeta}. \tag{19}$$

Here a_{nr} are known real constants, and by changing the order of the summations in (18) it follows that w' can be interpreted as the w_0' of (15), if it is assumed that

$$\gamma_n = -i \sum_{r=1}^{m} a^r H_r a_{rn}. \tag{20}$$

Again when $\xi > 0$, a complex potential can be introduced in region (3) given by

$$w = \frac{i\mu_3 I\zeta}{2\pi} + i\mu_3 \sum_{n=1}^{\infty} \left\{ \alpha_n' e^{n\zeta} + \left(\beta_n' - \frac{I_1 + I_2}{2\pi n} \right) e^{-n\zeta} \right\}, \tag{21}$$

where A_3 is the imaginary part of w, if by comparison with (11),

$$\alpha_n' = B_n - iC_n, \quad \beta_n' = F_n + f_n + i(G_n + g_n). \tag{22}$$

At large distances from the cylinders in the region where $\xi > 0$, the part of the complex potential corresponding to the external magnetic field can be written as

$$w_0' = i\mu_3 \sum_{n=1}^{\infty} \gamma_{nc}' e^{-n\xi},$$

where from (9) it follows that $\gamma_n' = f_n - ig_n$, and $\gamma_{nc}' = f_n + ig_n$. In terms of the coefficients of (16), with $\xi > 0$, it can similarly be proved that

$$\gamma_n' = i \sum_{r=1}^{m} (-1)^r a^r H_{rc} a_{rn}, \tag{23}$$

where the a_{rn} are defined by (19), and H_{rc} is the complex conjugate of H_r.

Returning to (11), the arbitrary constants in the solutions to the field equations must be chosen so that the boundary conditions given by (5) at $\xi = \xi_1$ and $\xi = \xi_2$, respectively, are satisfied for all values of η in the range $0 \leqslant \eta \leqslant 2\pi$. This involves equating (i) the constant terms, (ii) terms involving $\sin n\eta$, and (iii) terms involving $\cos n\eta$, on each side of the boundary condition equations.

From (i) it can be deduced that only one of the constants B, B', B'' can be chosen arbitrarily, and $C = (I_2 - I_1)/4\pi$. The relations resulting from (ii) and (iii) can be solved simultaneously to give the values of the remaining constants. In particular, if the notation is simplified by substituting

$$\lambda_1 = \frac{\mu_3 - \mu_1}{\mu_3 + \mu_1}, \quad \lambda_2 = \frac{\mu_3 - \mu_2}{\mu_3 + \mu_2}, \quad \nu_n = 1 - \lambda_1\lambda_2(x_1x_2)^n,$$

where $x_1 = e^{2\xi_1}$, and $x_2 = e^{-2\xi_2}$, then (14) and (22) reduce to

$$\nu_n\alpha_n = \gamma_n - \lambda_2\gamma_n'x_2^n + \frac{x_2^n}{2\pi n}\{I_2 + \lambda_2 I_1 - \lambda_2 x_1^n(I_1 + \lambda_1 I_2)\},$$

$$\nu_n\beta_n = \lambda_1(\lambda_2\gamma_{nc}'x_2^n - \gamma_{nc})x_1^n + \frac{x_1^n}{2\pi n}\{I_1 + \lambda_1 I_2 - \lambda_1 x_2^n(I_2 + \lambda_2 I_1)\},$$

$$\nu_n\alpha_n' = \lambda_2(\lambda_1\gamma_n x_1^n - \gamma_n')x_2^n + \frac{x_2^n}{2\pi n}\{I_2 + \lambda_2 I_1 - \lambda_2 x_1^n(I_1 + \lambda_1 I_2)\},$$

$$\nu_n\beta_n' = \gamma_{nc}' - \lambda_1\gamma_{nc}x_1^n + \frac{x_1^n}{2\pi n}\{I_1 + \lambda_1 I_2 - \lambda_1 x_2^n(I_2 + \lambda_2 I_1)\}. \tag{24}$$

As the vector potential components A_1 and A_2 are not required in the

subsequent analysis their values are not recorded here, although the constants involved in their expressions are readily determined.

The functions α_n, β_n, α_n', β_n', in (24) can be determined if the currents in the cylinders, the permeabilities of the media concerned, and the geometry of the system of cylinders are known. If $2d$ is the distance between the centers of the two circular cylinders of radii R_1 and R_2, respectively, then

$$x_1^{\frac{1}{2}} = \{(a^2 + R_1^2)^{\frac{1}{2}} - a\}/R_1, \quad x_2^{\frac{1}{2}} = \{(a^2 + R_2^2)^{\frac{1}{2}} - a\}/R_2,$$

where

$$a^2 = \{4d^2 - (R_1 + R_2)^2\}\{4d^2 - (R_1 - R_2)^2\}/16d^2.$$

The complex potential functions representing the magnetic field at all points external to the cylinders are thus completely determined.

V. THE MECHANICAL FORCES ON THE CYLINDERS

The force $\mathbf{F}$ acting on a cylindrical body set in a fluid medium has been deduced by Stratton, (7, p. 155), and in a more direct manner by Brown (9), in the form

$$\mathbf{F} = \mu \int_c \{\mathbf{H}(\mathbf{H}\cdot\mathbf{n}) - \tfrac{1}{2}H^2\mathbf{n}\}ds,$$

where $\mathbf{H}$ is the magnetic intensity measured along a curve c, near but external to the contour of the cylinder section, $\mathbf{n}$ is the unit outward-drawn normal to the curve c, and μ is the inductive capacity of the region outside the cylinder. If $\mathbf{F}$ has components X, Y in the respective axial directions in the z-plane, and $\mathbf{n}$ has components $n_x = dy/ds$, $n_y = -dx/ds$, then use of (12) leads to the result

$$Y + iX = \frac{1}{2\mu_3}\int_c \left(\frac{dw}{dz}\right)^2 dz. \tag{25}$$

This differs from the formula deduced by Power (10), for the analogous result in dielectric media, where the effect of electrostriction in the media was neglected.

If the curve c is considered to be the curve corresponding to $\xi = \text{const.}$ $(>\xi_1)$, surrounding the cylinder defining region (1), but not enclosing any of the singularities of the external magnetic field, then (25) can be interpreted as an integral in ζ, where, as $\xi < 0$, w is defined by (13), and from (17),

$$\left(\frac{dz}{d\zeta}\right)^{-1} = \frac{e^{\zeta} + e^{-\zeta} - 2}{2a}.$$

On the curve c it is known that $d\zeta = id\eta$, hence ζ changes by $2\pi i$ in describing the curve c once in an anticlockwise direction, and since the

only contribution to the value of the integral (25) is due to the constant term in the integrand, then

$$Y_1 + iX_1 = -\frac{i\mu_3}{4\pi a}\left[I_1I_2 - 2\pi(I_1\alpha_1 + I_2\beta_1) + 4\pi^2\sum_{n=1}^{\infty}\{2n^2\alpha_n\beta_n - n(n+1)(\alpha_{n+1}\beta_n + \alpha_n\beta_{n+1})\}\right], \quad (26)$$

where α_n, β_n are given by (24).

In order to deduce the force on the cylinder corresponding to $\xi = \xi_2$, it is necessary to take w in the form given by (21) for $\xi > 0$. In this case in describing the appropriate contour c in an anticlockwise sense η varies from 2π to 0, hence ζ changes by $-2\pi i$. It follows that for this cylinder the components of the force acting on it are given by

$$Y_2 + iX_2 = \frac{i\mu_3}{4\pi a}\left[I_1I_2 - 2\pi(I_1\alpha_1' + I_2\beta_1') + 4\pi^2\sum_{n=1}^{\infty}\{2n^2\alpha_n'\beta_n' - n(n+1)(\alpha_{n+1}'\beta_n' + \alpha_n'\beta_{n+1}')\}\right], \quad (27)$$

where α_n', β_n' are given by (24).

The torque on a cylinder has a moment L about the mid-point O of PQ, that is, the origin of coordinates in the z-plane, given by

$$\mathbf{L} = \mu\int_c \{(\mathbf{H}\cdot\mathbf{n})(\mathbf{r}\times\mathbf{H}) - \tfrac{1}{2}H^2(\mathbf{r}\times\mathbf{n})\}ds$$

where $\mathbf{r}$ is the radius vector drawn from O to the point (x,y) in the plane of the curve c. In terms of w this reduces to

$$L = \mathrm{Re}\,\frac{1}{2\mu_3}\int_c\left(\frac{dw}{dz}\right)^2 z\,dz.$$

Proceeding in the same manner as for the determination of the force, with

$$z\left(\frac{dz}{d\zeta}\right)^{-1} = \frac{e^{-\zeta} - e^{\zeta}}{2},$$

from (17), it can readily be shown that

$$L_1 = \mathrm{Re}\,\frac{i\mu_3}{2}\{I_1\alpha_1 - I_2\beta_1 + 2\pi\sum_{n=1}^{\infty} n(n+1)(\alpha_{n+1}\beta_n - \alpha_n\beta_{n+1})\}, \quad (28)$$

and

$$L_2 = \mathrm{Re}-\frac{i\mu_3}{2}\{I_1\alpha_1' - I_2\beta_1' + 2\pi\sum_{n=1}^{\infty} n(n+1)(\alpha_{n+1}'\beta_n' - \alpha_n'\beta_{n+1}')\}. \quad (29)$$

Here L_1 and L_2 are the moments of the torque acting on the cylinders forming regions (1) and (2), respectively, both taken about the point O. The forces and torques acting on both cylinders are thus completely determined in terms of known constants.

VI. SOME PARTICULAR EXAMPLES

(i) From the engineering point of view, simplicity of application of formulas is a criterion of prime importance. Such formulas must give a fair representation of the physical facts. Most materials used as conductors have inductive capacities which are almost equal to that of free space, hence to a close approximation in this instance it can be assumed that $\lambda_1 = \lambda_2 = 0$. If this is the only restriction then (26) reduces to

$$Y_1 + iX_1 = -\frac{i\mu_2 I_1 I_2}{4\pi d} + \mu_2 I_1 \sum_{n=1}^{\infty} nH_n d_1^{n-1}, \tag{30}$$

where use has been made of the relations

$$\frac{a}{d} = \frac{(1 - x_1)(1 - x_2)}{1 - x_1 x_2}, \tag{31}$$

and

$$d_1 = \frac{a(1 + x_1)}{1 - x_1} = \frac{4d^2 - R_2^2 + R_1^2}{4d}. \tag{32}$$

Here d_1 is the distance of the center of the cylinder enclosing region (1) from O.

Similarly (27) reduces to

$$Y_2 + iX_2 = \frac{i\mu_2 I_1 I_2}{4\pi d} - \mu_2 I_2 \sum_{n=1}^{\infty} (-1)^n nH_n d_2^{n-1}, \tag{33}$$

where

$$d_2 = \frac{4d^2 - R_1^2 + R_2^2}{4d}, \tag{34}$$

being the distance of the center of the cylinder enclosing region (2) from the point O.

When the external magnetic field is present the resultant force on each cylinder does not in general act along the line of centers of the cylinders. However in the absence of this field the force between the cylinders acts along the line of centers, and if $I_1 = I_2$ the force is one of attraction, whereas if $I_1 = -I_2$ the force is one of repulsion. These are well-known results.

The torques on the cylinders have magnitudes $L_1 = d_1 Y_1$, and $L_2 = -d_2 Y_2$, hence even when there is an external magnetic field present the moment of the resultant forces about the center of each cylinder are respectively zero.

(ii) If the cylinders are at such a distance apart that the ratios d/R_1 and d/R_2 are large, then to a second approximation, in the absence of the external magnetic field,

$$X_1 = -\frac{\mu_2}{4\pi d}\left(I_1 I_2 - \frac{\lambda_1 R_1^2 I_2^2 + \lambda_2 R_2^2 I_1^2}{4d^2}\right), \tag{35}$$

and $Y_1 = L_1 = 0$. The force on the second cylinder is such that $X_2 = -X_1$, and $Y_2 = L_2 = 0$. The number of terms retained when the external field is present will depend on the relative magnitudes of the coefficients H_n. If $I_1 = I_2$ the second order term has the effect of reducing the attractive force between the cylinders when both the cylinders are formed of diamagnetic material, ($\lambda > 0$), and of increasing the force if the material is paramagnetic, ($\lambda < 0$). An opposite effect occurs when $I_1 = -I_2$, the force then being one of repulsion.

(iii) Another example of interest is to estimate the second order effect of the variation in the permeabilities of the materials forming the conductors. It will be sufficient in this case to consider the two cylinders identical in all respects, that is, $\lambda_1 = \lambda_2 = \lambda$, and $x_1 = x_2 = x$. In particular if $I_1 = -I_2 = I$, that is, the cylinders form a return circuit, then in the absence of the external field the repulsive force between the cylinders can be written in the form

$$X = \frac{\mu_2 I^2}{4\pi d}\left\{1 + 2\left(\frac{1+x}{1-x}\right)\sum_{n=1}^{\infty}\lambda^n\left[\frac{nx^n}{1-x^n} - \frac{2(n+1)x^{n+1}}{1-x^{n+1}} + \frac{(n+2)x^{n+2}}{1-x^{n+2}}\right]\right\}. \tag{36}$$

This of course can be extended to the case when the effect of the external magnetic field is included, and also when the cylinders are not identical, but as these expressions are lengthy they are omitted here. The second order effect of non-zero λ is deduced by neglecting powers of λ greater than the first. Although perhaps of little practical value the force between the cylinders when they are in contact reduces to

$$X = \frac{\mu_2 I^2}{4\pi d}\left\{1 + \frac{2\lambda}{3(1-\lambda)}\right\}.$$

If $D = d/R$, where R is the radius of either cylinder, and $F = 4\pi XR/\mu_2 I^2$, then for a given set of cylinders, X can be determined for distances of separation in the region $R \leqslant d < \infty$. Thus for values of $\lambda = -0.2$, 0, 0.2, values of F can be deduced from Table I.

TABLE I.—*The Variation of F.*

λ	$D =$ 1	1.5	2	5	10
−0.2	0.889	0.635	0.487	0.199	0.100
0	1.000	0.667	0.500	0.200	0.100
0.2	1.167	0.702	0.514	0.201	0.100

When the conducting material is paramagnetic, ($\lambda = -0.2$), the effect is to decrease the repulsive force as compared with non-magnetic conducting material, ($\lambda = 0$), and to increase the force when the material is diamagnetic, ($\lambda = 0.2$). This effect is more pronounced at small distances of separation, and is of small order at large separation distances.

When the currents are such that $I_1 = I_2 = I$, the force X is an attractive force and has a magnitude found by changing the sign of λ in (36), and consequently give an inverse effect to the conclusions drawn from Table I.

(iv) When the external magnetic field is present there is in general a force acting on each cylinder normal to the line of centers of the cylinders. If the cylinders are identical and $I_1 = -I_2 = I$, and the external field has uniform intensity H at an angle α to the line of centers, this force has the comparatively simple form

$$Y_1 = \mu_2 HI\left(\frac{1-x}{1+x}\right)\left\{1 + 2(1-\lambda)\sum_{n=1}^{\infty}\frac{x^n}{1-\lambda x^n}\right\}\cos\alpha$$

$$+ 8\pi a\mu_2\lambda^2 H^2\left(\frac{1-x}{1+x}\right)\sum_{n=1}^{\infty}\frac{nx^{2n}}{1-\lambda^2 x^{2n}}\sin 2\alpha,$$

and $Y_2 = -Y_1$. If the cylinders were rigidly connected together the system would be acted upon by a torque of moment $L_1 + L_2 = 2dY_1$.

In conclusion it can be pointed out that the theory developed in this paper can equally well be adapted to the set up where there exists a cylindrical conductor whose external and internal surfaces coincide, respectively, with two cylinders of the same coaxal system, the current flowing in the material between the two bounding surfaces. Again this can also be extended to the system of two such conductors set parallel to each other, and the forces can be determined by use of formulas involving contour integrals.

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TIME DEPENDENT FORMULATION OF THE COLLISION PROBLEM IN QUANTUM MECHANICS*

BY

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1. INTRODUCTION

The treatment of the collision problem in the standard text books[2] on quantum mechanics is often felt by the perceptive student to be rather unsatisfactory. Indeed the solution of the problem as presented in the texts suffers a number of deficiencies. (1) The solution employs elements which are foreign to orthodox particle quantum mechanics in the use of (*a*) non-normalized eigenfunctions, and (*b*) the "probability current" as a measure of particle flow. Generally the student is not bothered by these features so much as by (2) the apparent disparity between the form of the solution given and the actual experimental situation in the laboratory. The incident beam which has a finite cross section in the laboratory is described by an infinite plane wave. This leads to an interference between the incident wave and the "scattered wave" and the separation of the two waves in the standard solution of the problem appears to be arbitrary. Another feature which is generally felt to be unsatisfactory is (3) that the text book solution provides no picture of the time development of the collision process. This is not, in principle, a fault since the collision problem certainly should be capable of formulation as a steady state involving a steady stream of incident particles. However, the relation between the text book solution and a time dependent picture of the process is not provided and remains obscure to the student.

In the treatment of the collision problem formulated below we obtain a time dependent solution of the problem in which normalized state functions are employed, no appeal to probability current is made, and a reasonably close fit to the experimental initial conditions is employed. The procedure, of course, fully justifies the far shorter text book solutions.

2. GENERAL THEORETICAL FORMULATION

We shall suppose, for simplicity, that the collision is between two structureless particles, the interaction between which is described by a

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[1] Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

[2] For example: D. Bohm, "Quantum Theory," Prentice-Hall, 1951; L. I. Schiff, "Quantum Mechanics," McGraw-Hill, 1949; N. F. Mott and H. S. W. Massey, "The Theory of Atomic Collisions," Oxford University Press, 1933.

centrally symmetric potential $V(r)$. The generalization of the considerations below to more complex cases is not difficult. If the initial state of the system is described by a wave function, $\Psi(0)$, the wave function at later times is given by that solution of

$$H\Psi = i\hbar \frac{\partial \Psi}{\partial t} \tag{1}$$

which reduces to $\Psi(0)$ at $t = 0$; H in Eq. 1 is the Hamiltonian for the system. A formal solution of Eq. 1 is

$$\Psi(t) = \sum c_k \Psi_k e^{-iE_k t/\hbar}, \qquad \Psi(0) = \sum c_k \Psi_k \tag{2a}$$

$$c_k = (\Psi_k, \Psi(0)) \tag{2b}$$

where the Ψ_k are normalized eigenfunctions of H belonging to the eigenvalues E_k. The c_k are defined through Eq. 2*b*. If the spectrum of H is continuous so that its eigenfunctions are defined by a continuous parameter (or several such parameters) the solution has the form

$$\Psi(t) = \int c(p)\Psi(p)e^{-iE(p)t/\hbar}dp \tag{2c}$$

$$\Psi(0) = \int c(p)\Psi(p)dp, \tag{2d}$$

where

$$H\Psi(p) = E(p)\Psi(p). \tag{2e}$$

In the Hamiltonian for the collision problem it is useful to introduce, instead of the coordinates of the two colliding particles, a relative coordinate, $\mathbf{r}$, and a center of mass coordinate, $\mathbf{R}$, together with their conjugate momenta. H then takes the form

$$H = \frac{\mathbf{P}^2}{2M} + \frac{\mathbf{p}^2}{2m} + V(\mathbf{r}),$$

where $\mathbf{P}$ and $\mathbf{p}$ are the momenta conjugate to the coordinates $\mathbf{R}$ and $\mathbf{r}$; M is the total mass and m the reduced mass for the system. The Hamiltonian separates into a part H_R for the center of mass motion ($H_R = \mathbf{P}^2/2M$) and a part H_r describing the relative motion. In consequence the eigenfunctions of H may be obtained as products of eigenfunctions of H_R and H_r, respectively. The eigenfunctions of H_R are just the free particle eigenstates:

$$H_R\chi_{\mathbf{P}'}(\mathbf{R}) = E_R(\mathbf{P}')\chi_{\mathbf{P}'}(\mathbf{R}), \tag{3a}$$

where

$$\chi_{\mathbf{P}'}(\mathbf{R}) = Ae^{i\mathbf{P}'\cdot\mathbf{R}/\hbar}, \tag{3b}$$

and

$$E(P') = \frac{\mathbf{P}^{12}}{2M}. \tag{3c}$$

As is indicated below the eigenstates of H_r for E_r, the energy of relative motion, greater than zero, can be designated by a vector parameter $\mathbf{k}$. For the moment we write simply

$$H\phi_{\mathbf{k}}(\mathbf{r}) = E(\mathbf{k})\phi_{\mathbf{k}}(\mathbf{r}). \tag{4}$$

Eigenstates for the full Hamiltonian are thus

$$H\phi_{\mathbf{k}}\chi_{\mathbf{P}'} = (E(\mathbf{k}) + E(\mathbf{P}'))\phi_{\mathbf{k}}\chi_{\mathbf{P}'}. \tag{5}$$

If the initial state of the system can be written as a product of a relative motion state ϕ and a center of mass state χ, that is,

$$\Psi(\mathbf{r},\mathbf{R},0) = \phi(\mathbf{r},0)\chi(\mathbf{R},0) \tag{6}$$

then, as can be seen by substitution into Eq. 1, at any later time

$$\Psi(\mathbf{r},\mathbf{R},t) = \phi(\mathbf{r},t)\chi(\mathbf{R},t), \tag{6a}$$

where ϕ and χ are the solutions of the separate time dependent equations

$$H_r\phi = i\hbar\frac{\partial\phi}{\partial t} \quad H_R\chi = i\hbar\frac{\partial\chi}{\partial t} \tag{6b}$$

which reduce to $\phi(\mathbf{r},0)$ and $\chi(\mathbf{R},0)$ at $t = 0$. Since we shall see that we may, with reasonable approximation, restrict ourselves to initial states of the type (6), we shall need solutions of the separate time dependent equations (6*b*).

The general solution for $\chi(\mathbf{R},t)$ is easily written:

$$\chi(\mathbf{R},t) = \int A(\mathbf{P})e^{i\mathbf{P}\cdot\mathbf{R}/\hbar}e^{-iE(P)t/\hbar}d\mathbf{P}, \tag{7}$$

where $A(\mathbf{P})$ is an arbitrary function of the vector $\mathbf{P}$. The $A(\mathbf{P})$ is chosen so that $\chi(\mathbf{R},0)$ describes the part of the initial state referring to the center of mass. The function $|A(\mathbf{P})|^2$ describes the (relative) probability distribution of the momentum $\mathbf{P}$ of the center of mass motion. This momentum distribution remains constant in time. The state function $\chi(\mathbf{R},t)$ is normalizeable if $|A(\mathbf{P})|^2$ is square integrable.

3. A GENERAL DESCRIPTION OF THE RELATIVE COORDINATE STATES

The Schrödinger equation for the energy eigenfunctions of H_r is

$$\left(-\frac{\hbar^2}{2m}\nabla_r^2 + V(r)\right)\phi(\mathbf{r}) = E\phi(\mathbf{r}). \tag{8}$$

For large values of r, the $V(r)$ is negligible (we assume for simplicity that $V(r) \to 0$ at least as fast as r^{-2}) so that the equation becomes, asymptotically,

$$(\nabla_r^2 + k^2)\phi(\mathbf{r}) = 0 \quad (r \text{ large}), \tag{8a}$$

where

$$k^2 = \frac{2mE}{\hbar^2}. \tag{8b}$$

Particular solutions of (8a), but not of (8) are

$$e^{i\mathbf{k}\cdot\mathbf{r}} \quad \text{and} \quad r^{-1}e^{\pm ikr}g_\pm(\theta,\phi), \tag{9}$$

where in the "plane wave" solution, $e^{i\mathbf{k}\cdot\mathbf{r}}$, $\mathbf{k}$ may have an arbitrary direction, and in the "spherical wave" solutions, the $g_\pm(\theta,\phi)$ functions may be arbitrary. Both the solutions of Eqs. 8a and of (8), it may be noted, are infinitely degenerate. It is shown in the text books on quantum mechanics that solutions of (8) may be taken in such form that they become, for large values of r,

$$\phi_\mathbf{k}(\mathbf{r}) \to e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta,\phi,\mathbf{k})r^{-1}e^{ikr} \tag{10}$$

where the function $f(\theta,\phi,\mathbf{k})$, which depends on the form of the potential $V(r)$, is completely determined by Eq. 8. The arrow instead of the equality indicates that ϕ takes the given form asymptotically (that is, for $r \to \infty$). The details of $\phi_\mathbf{k}(\mathbf{r})$ for small values of r are not needed, fortunately, for our subsequent analysis. Of course, the independent eigenfunctions of Eq. 8 may be chosen in a great variety of ways. The form employed above is explicitly adapted to the collision problem as will be seen below.

The asymptotic form of the time dependent solution can now be given:

$$\phi(\mathbf{r},t) \to \int B(\mathbf{k})(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta,\phi,\mathbf{k})r^{-1}e^{ikr})e^{-iE(k)t/\hbar}d\mathbf{k}, \tag{11a}$$

where $B(\mathbf{k})$ may be chosen arbitrarily. To obtain a particular solution with given $\phi(\mathbf{r},0)$ at $t = 0$, the $B(\mathbf{k})$ must be chosen so that

$$\phi(\mathbf{r},0) \to \int B(\mathbf{k})(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta,\phi,\mathbf{k})r^{-1}e^{ikr})d\mathbf{k}. \tag{11b}$$

4. THE INITIAL CONDITION OF THE COLLISION SYSTEM

We shall assume that the collision experiment is performed under the following conditions. A source, slit, and shutter system are arranged so that, at $t = 0$, the incident particle is known to be within a

small (but macroscopic) cylinder on the z axis in the neighborhood of a position $\mathbf{r}_0$ (as measured from the mean position of the target particle) where $\mathbf{r}_0$ has the components $(0, 0, -z_0)$ and z_0 is positive; the average value of the initial momentum is along the positive z axis (that is, directed toward the target) and there are small uncertainties in the initial vector momentum. In principle the apparatus could be refined until the uncertainties in the momentum components are reduced to the limits imposed by the Heisenberg uncertainty relations (for the given uncertainties in the position coordinates). For any reasonable uncertainty in the initial position the minimum uncertainties in the initial momentum components are extremely small. The minimum uncertainty in the initial kinetic energy (in ev ≡ electron volts) may be shown to be $\sim 10^{-9}\sqrt{E_0(\text{ev})}$, where E_0 is the average value of this initial energy (in electron volts). The minimum uncertainty in the direction is $\sim 10^{-8}/\sqrt{E_0(\text{ev})}$ degrees. (These estimates are based on the assumption that the particles have a mass equal to that of a nucleon and that the uncertainty in position is of the order of 1 cm.) In general the uncertainties in the momentum components introduced by the experimental arrangements are considerably larger. The target particle is assumed to be at rest, or more accurately to have an average momentum equal to zero and a small (but macroscopic) positional uncertainty. Again the actual momentum distribution of the target is not precisely defined. The minimum uncertainty in momentum corresponds to an energy uncertainty of the order of only 10^{-15}(ev) which is, of course, entirely negligible.

The experimental situation does not specify a definite initial state of the system. There is indeed a great multiplicity of different states which are consistent with the somewhat vague conditions outlined above. Under these circumstances one should calculate the collision cross section for the various initial states which are consistent with the experimental restrictions and then, in some manner, average the results. We shall see, however, that as long as the uncertainties in the initial momenta are made sufficiently small, the collision cross section is effectively the same for all the initial states which are possible.

We may construct an initial state which is consistent with the experimental restrictions as follows. We shall take it in the form

$$\phi(\mathbf{r},0)\chi(\mathbf{R},0),$$

where $\phi(\mathbf{r},0)$ describes a packet for the relative motion which initially has a macroscopic uncertainty in $\mathbf{r}$ about $\mathbf{r} = \mathbf{r}_0$, an average relative momentum which is the average initial relative momentum defined by the apparatus, and an uncertainty in this relative momentum which is larger than that specified by the Heisenberg relations but "sufficiently

small" in a sense to be elaborated later. $\chi(\mathbf{R},0)$ is to be similarly defined. Thus $\chi(\mathbf{R},0)$ is given by

$$\chi(\mathbf{R},0) = \int A(\mathbf{P}) e^{i(\mathbf{P}\cdot\mathbf{R})/\hbar} d\mathbf{P} \tag{12}$$

and $A(\mathbf{P})$ is to be chosen so that $|A(P)|^2$ has a definite second moment (uncertainty) equal to the experimentally determined $\Delta\mathbf{P}$ for the center of mass motion and such that the average value of the position of the center of mass is that defined by the apparatus. The $\phi(\mathbf{r},0)$ may also be expanded in terms of the eigenfunctions of the Hamiltonian H_r. Since the initial state of the system is such that $\phi(\mathbf{r},0)$ is large only for large values of r, we may use the asymptotic forms of the eigenfunctions.

$$\phi(\mathbf{r},0) = \int B(\mathbf{k})(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta,\phi,\mathbf{k})r^{-1}e^{ikr})d\mathbf{k}. \tag{13}$$

We must choose $B(\mathbf{k})$ so that $\phi(\mathbf{r},0)$ represents a packet different from zero only about $\mathbf{r} = \mathbf{r}_0$ with an average initial momentum $\mathbf{p}_0$ (with components $(0,0,\mathbf{p}_0)$) and with "sufficiently small" uncertainties in this initial momentum. We shall refer to the two parts of the integrand of Eq. 13 as the "plane wave part" and the "spherical part," respectively.

The eigenfunctions of the relative momentum are $e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}$ where $\mathbf{p}$ designates the momentum eigenvalue. Thus the plane wave part of Eq. 13 is recognized as a linear combination of pure momentum states with

$$\mathbf{p} = \hbar\mathbf{k}. \tag{14}$$

Since, further, the initial state of the relative motion is free, we may expect that $B(\mathbf{k})$ should be chosen so that only the plane wave part of Eq. 13 contributes to $\phi(\mathbf{r},0)$ and that $|B(\mathbf{k})|^2$ defines effectively the distribution in $\mathbf{k}$ and hence in $\mathbf{p}$ in the initial state of the system. With these considerations in mind, we try to form $\phi(\mathbf{r},0)$ using a $B(\mathbf{k})$ such that the uncertainty in $|B(\mathbf{k})|^2$, that is, Δk, is just $\Delta\mathbf{k} = \Delta\mathbf{p}/\hbar$.

The position of the maximum in the amplitude of $\phi(\mathbf{r},0)$ may be obtained by the "method of stationary phase." We write

$$B(\mathbf{k}) = b(\mathbf{k})e^{i\beta(\mathbf{k})} \tag{15}$$

with $b(\mathbf{k})$ and $\beta(\mathbf{k})$ real. For the plane wave part of Eq. 13 we have

$$\int b(\mathbf{k})e^{i(\beta(\mathbf{k})+\mathbf{k}\cdot\mathbf{r})}d\mathbf{k}$$

which has a maximum for that value of $\mathbf{r}$ at which the phase of the integrand is stationary at $\mathbf{k} = \mathbf{k}_0 = \mathbf{p}_0/\hbar$, where $\mathbf{p}_0$ is the average initial

momentum. Thus $\mathbf{r}_m(0)$, the position of the maximum amplitude of the integral above, is given by that $\mathbf{r}$ which satisfies

$$\frac{\partial}{\partial k_i}(\beta(\mathbf{k}) + \mathbf{k}\cdot\mathbf{r})\Big|_{\mathbf{k}_0} = 0, \tag{16}$$

that is,

$$\mathbf{r}_m(0) = -\nabla_k\beta(\mathbf{k}). \tag{16a}$$

If we set the position of the maximum amplitude equal to the initial mean position $(0, 0, -z_0)$ of the particle we have

$$\frac{\partial\beta(\mathbf{k})}{\partial k_1}\Big|_{\mathbf{k}_0} = \frac{\partial\beta(\mathbf{k})}{\partial k_2}\Big|_{\mathbf{k}_0} = 0, \quad \frac{\partial\beta(\mathbf{k})}{\partial k_3}\Big|_{\mathbf{k}_0} = z_0. \tag{17}$$

In the spherical part of Eq. 13, r takes on only positive values. The rate of change of phase of the integrand for this part at $\mathbf{k} = \mathbf{k}_0$ is

$$\nabla_k(\beta(\mathbf{k}) + kr) = \nabla_k\beta(\mathbf{k})\,|_{\mathbf{k}_0} + \boldsymbol{\varepsilon}_3 r$$

($\boldsymbol{\varepsilon}_3$ is a unit vector along the z axis). Using the $\nabla_k\beta(\mathbf{k})$ as given by Eq. 17 we find that the phase of the spherical part of Eq. 13 is not stationary for any positive value of r. Indeed the rate of change of phase for all positive r is very large in the neighborhood of $\mathbf{k} = \mathbf{k}_0$ so that the amplitude of the contribution from this part to $\phi(\mathbf{r},0)$ is negligibly small. Thus, at $t = 0$,

$$\phi(\mathbf{r},0) \cong \int B(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{k},$$

where the function $B(\mathbf{k})$ is as yet only partially determined. The $\Delta\mathbf{k} = \Delta\mathbf{p}/\hbar$ of $|B(\mathbf{k})|^2$ is given, and $\nabla\beta(\mathbf{k}_0)$ is fixed (by Eq. 17). Any choice of $B(\mathbf{k})$ consistent with these restrictions is permitted. The remaining details of $B(\mathbf{k})$ will determine the form of the spatial distribution of the initial packet.

It should be noted that $B(\mathbf{k})$ is chosen so that its rate of change of phase at $\mathbf{k}_0$ is very large. This phase change is made to balance the rapid phase change of $e^{i\mathbf{k}\cdot\mathbf{r}}$ for $\mathbf{r} = \mathbf{r}_0$. For the smaller values of $\mathbf{r}$ where ϕ_k is not defined by our asymptotic forms the phase of $\phi_k(r)$ may be shown to vary much less rapidly with $\mathbf{k}$. Indeed the phase at all positions varies with k roughly as kr, at least in magnitude. Hence for small values of $\mathbf{r}$, we get negligible contributions to $\phi(\mathbf{r},0)$ in Eq. 13.

5. THE MOTION OF THE RELATIVE COORDINATE PACKET

Once the $B(\mathbf{k})$ has been defined the state of relative motion at all later times, $\phi(\mathbf{r},t)$, is given by Eq. 11a at least for sufficiently large values of r. We can again find the values of $\mathbf{r}$ and t at which the

amplitude of ϕ is a maximum by the method of stationary phase. We get large contributions to $\phi(\mathbf{r},t)$ if either of the relations

$$\nabla_k[\beta(\mathbf{k}) + \mathbf{k}\cdot\mathbf{r} - Et/\hbar]_{k=k_0} = (z_0 - p_0 t/m)\mathbf{e}_3 + \mathbf{r} = 0; \tag{18a}$$

$$\nabla_k[\beta(\mathbf{k}) + kr + \alpha(\theta,\phi,\mathbf{k}) - Et/\hbar]_{k=k_0} = [z_0 + r - p_0\hbar/m]\mathbf{e}_3 + (\nabla_k\alpha)_{k=k_0} = 0, \tag{18b}$$

where $\alpha(\theta,\phi,\mathbf{k})$ is the phase of the function $f(\theta,\phi,\mathbf{k})$ and $\mathbf{e}_3$ is a unit vector along the z axis. (We have here used

$$E(\mathbf{k}) = \hbar^2k^2/2m, \quad \nabla_k E/\hbar\Big|_{k_0} = \mathbf{k}_0\hbar/m = \mathbf{p}_0/m = \frac{p_0}{m}\mathbf{e}_3.)$$

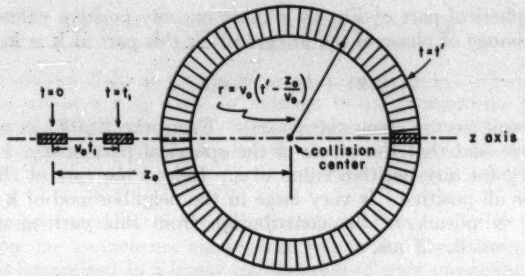

FIG. 1. The motion of the relative coordinate packet. The form of the relative coordinate packet at $t = 0$, at "small" times, t_1, and at large times, t'. At large t the packet has two interfering contributions on the z axis from the plane and spherical wave-contributions to the solution.

Thus, from the plane wave part we get strong contributions to $\phi(\mathbf{r},t)$ only in the neighborhood of

$$\mathbf{r} \cong \left(\frac{p_0}{m}t - z_0\right)\mathbf{e}_3, \tag{19}$$

that is, only for positions in the neighborhood of the z axis. Note that for small t, this relation describes a uniform motion of the maximum of the packet with velocity $\mathbf{v} = v_0\mathbf{e}_3$, where $v_0 = p_0/m$, the classical velocity associated with the average momentum p_0. For $t \gg z_0/v_0$ we have the maximum of the packet still in the neighborhood of the z axis and moving with velocity v_0. Indeed, the plane wave part behaves exactly like a free particle packet. The contributions to $\phi(\mathbf{r},t)$ are large for the spherical part if Eq. 18b holds, that is, if

$$r \cong v_0 t - z_0 - [\mathbf{e}_3\cdot(\nabla\alpha)]_{k=k_0}. \tag{20}$$

Now an investigation of the solutions (Eq. 10) show that $\nabla\alpha$ is a *microscopic* quantity. It is a length which is of the order of the range of the potential $V(r)$. Hence the last term in Eq. 20 may be dropped since we are here interested only in macroscopic distances. Since r in Eq. 20 is positive, this equation is not satisfied unless $t > z_0/v$. Also since all our considerations hold only for r large, we have a description of the large contributions to $\phi(\mathbf{r},t)$ only for $t \gg z_0/v$. Note that the condition above is independent of θ, ϕ. From the results given by Eqs. 19 and 20, the character of the motion of the packet for $v_0 t \ll z_0$ and for $v_0 t \gg z_0$ is easily seen. (See Fig. 1.) Initially we have the plane wave packet moving along the positive z axis with uniform velocity. For large t we have in addition to the plane wave packet, a spherical packet diverging from the origin of the relative coordinate.[3] (The amplitude of the spherical packet is, of course, a function of direction.)

6. THE PROBABILITY OF SCATTERING AND THE DIFFERENTIAL CROSS SECTION

We consider the function $\phi(\mathbf{r},t)$ for $t \gg z_0/v$ so that the spherical packet is already in the region of large r. Except in the direction of the z axis, the wave function is given by

$$\phi(\mathbf{r},t) = \int B(\mathbf{k}) f(\theta,\phi,\mathbf{k}) r^{-1} e^{ikr} e^{-iEt/\hbar} d\mathbf{k} \quad (\theta \neq 0). \tag{21}$$

We assume that $B(\mathbf{k})$ has been chosen so that $\phi(\mathbf{r},0)$ is normalized to unity. Then $|\phi(\mathbf{r},t)|^2$ is the probability per unit volume of finding the relative coordinate $\mathbf{r}$ at time t. To obtain the probability that after the collision (that is, for t large) the relative coordinate has some value within a solid angle $d\Omega$ about a given direction θ, ϕ, we form

$$\mathcal{P}_* d\Omega = d\Omega \int_0^\infty |\phi(\mathbf{r},t)|^2 r^2 dr = d\Omega \int_0^\infty dr \left| \int B(k) f(\theta,\phi,\mathbf{k}) e^{ikr} e^{-iEt/\hbar} d\mathbf{k} \right|. \tag{22}$$

The integrand of the integral over $\mathbf{k}$ is important only for $\mathbf{k}$ in region Δk about $\mathbf{k}_0$. We now assume that Δk is small enough so that $f(\theta,\phi,\mathbf{k})$ may be considered as effectively constant over the range $\Delta\mathbf{k}$. Then we may write

$$\mathcal{P}_* = |f(\theta,\phi,\mathbf{k}_0)|^2 \int_0^\infty dr \left| \int B(\mathbf{k}) e^{ikr - iEt/\hbar} d\mathbf{k} \right|^2. \tag{22a}$$

Further, since the packet described by the integral over k is appreciably different from zero only for large positive values of r, the integration

[3] By an argument similar to that at the end of the last section, it may be seen that $\phi(\mathbf{r},t)$ for r not in the asymptotic region, at the small and large values of t under consideration above, are negligible.

over r may be formally extended, without changing its value to an integral over the range $\pm\infty$.

The probability $\mathcal{P}_s$ is not yet quite the cross section for scattering. The differential cross section is defined as the probability of scattering into unit solid angle divided by the probability per cm^2 that the particle is initially in a cylinder of unit cross section about the direction of the incident beam. The latter probability $\mathcal{P}_0$ is

$$\mathcal{P}_0 = \frac{1}{\mathfrak{a}} \int\int_{\mathfrak{a}} dxdy \int_{-\infty}^{\infty} dz |\phi(\mathbf{r},0)|^2 = \int_{-\infty}^{\infty} dz \left| \int B(\mathbf{k}) e^{ik_z z} d\mathbf{k} \right|^2, \quad (23)$$

where $\mathfrak{a}$ is the area of the base of a small cylinder about the z axis. We have used here the fact that the initial beam is presumed symmetric about the z axis and that $|\phi(\mathbf{r},0)|^2$ is effectively independent of x, y for small x, y. Only with such incident beams do we have a true measure of the differential cross section.

It is not difficult to see that the integrals in Eqs. 22a and 23 are equal. In Eq. 22a we may write

$$\exp(-iEt/\hbar) = \exp[-i(k_0^2 + 2k_0(k - k_0) + (k - k_0)^2\hbar t/2m)].$$

The factors which are independent of k may be taken out of the integral; when they are absolute squared they provide a factor unity. The $(k - k_0)^2\hbar t/2m$ does not exceed in magnitude $\sim (\Delta k)^2\hbar t/2m$ and for reasonable Δk and t values this term is very much smaller than 1 and may, in consequence, be neglected. The integral in Eq. 22a becomes

$$\int_{-\infty}^{\infty} dr \left| \int B(\mathbf{k}) e^{ik(r - \hbar k_0 t/m)} d\mathbf{k} \right|^2.$$

On changing the variable of integration to $\rho = r - \hbar k_0 t/m$ this reduces to

$$\int_{-\infty}^{\infty} d\rho \left| \int B(\mathbf{k}) e^{ik\rho} d\mathbf{k} \right|^2.$$

In Eq. 23, k_z may be replaced by k without changing the phase factor appreciably for laboratory magnitudes of z. It can readily be shown that k_z/k considered as a function of $\mathbf{k}$ has the form in the immediate neighborhood of $\mathbf{k} = \mathbf{k}_0$ (which is along the z axis)

$$k_z/k \cong 1 + \frac{3}{2}\left(\frac{k_z - k_0}{k_0}\right)^2.$$

Thus

$$e^{ik_z z} \cong e^{ikz} \cdot e^{i\frac{3}{2}k\left(\frac{k_z - k_0}{k_0}\right)^2 z}.$$

The phase of the last factor is, for $z = z_0$, that is, at the position of the maximum of the packet,

$$\frac{3}{2}\frac{(\Delta k)^2}{k_0} z_0.$$

For packets of the order of a centimeter in size, $\Delta k \sim 1$, and this phase is $\sim z_0$ (cm)$\cdot 10^{-9}/\sqrt{E_0(\text{ev})}$ where E_0 is the average energy of relative motion. For laboratory values of z_0 this is always $\ll 1$ and we may write

$$e^{ik_0 z} \cong e^{ikz}.$$

Thus the integral factors in $\mathcal{P}_s$ and $\mathcal{P}_0$ are equal, for the differential cross section and we have

$$\frac{d\sigma}{d\Omega} = \frac{\mathcal{P}_s}{\mathcal{P}_0} = |f(\theta,\phi,\mathbf{k}_0)|^2. \tag{24}$$

It may appear odd that these results depend on z_0 for the $\mathcal{P}_0$ calculation and t for the $\mathcal{P}_s$ calculation being macroscopic, but not too large. This is a consequence of the general phenomenon of the spreading of packets. If the packet is prepared too far away from the collision center the probability, per cm², in the initial packet is not equal to the probability, per cm², of being about the z axis at later times but before the relative coordinate is zero. In this case the $\mathcal{P}_0$ calculated at $t = 0$ is not the correct $\mathcal{P}_0$ for the collision problem. Similar considerations apply also to the motion of the packet after collision. For reasonable laboratory times and distances and Δp or Δk of the order of that given by the Heisenberg uncertainty relations, the approximations employed to provide the result (Eq. 24) are always extremely good. Effectively this condition implies that over laboratory times and distances, the spreading of the packet may be neglected.

It will be remembered (see Eq. 10 and preceding remarks) that the quantity $f(\theta,\phi,\mathbf{k}_0)$, the so-called scattering amplitude, is obtained from the solution of the time independent Schrödinger equation $H\phi = E\phi$ which for large values of r has the form

$$\phi \to e^{ik_0 z} + f(\theta,\phi,k_0) r^{-1} e^{ikr}, \tag{25}$$

where we have taken $\mathbf{k}_0$ along the z axis. Thus the cross section (Eq. 24) is known once this *particular eigenfunction* (Eq. 25) has been found. In the standard treatment of the collision problem it is argued that the eigenfunction (Eq. 25) represents the stationary state collision eigenfunction. The cross section (Eq. 24) is derived from the form of this solution. Both the stationary state and the time dependent methods lead, of course, to the same final result.

7. EFFECT OF THE CENTER OF MASS PACKET

The cross section above is for the change in direction of the relative coordinate. To obtain the cross section as a function of final direction of the incident particle, we must transform back from the relative and center of mass coordinates to the spatial coordinates of the two particles. To obtain the final direction of the incident particle we must add to the relative momentum the momentum of the center of mass. (See Fig. 2.) However $\mathbf{P}$, the center of mass momentum, is not precisely defined but has a distribution about some average value in the z direction ($\mathbf{P}_0$ along the z axis) which is defined by the initial condition of the system. The distribution of $\mathbf{P}$ is stationary in time since the

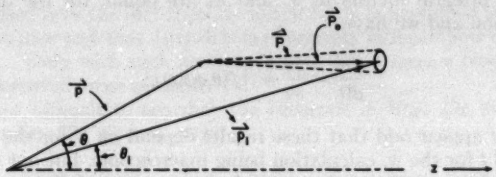

FIG. 2. The transformation to laboratory coordinates. Because of the uncertainty of the center of mass momentum, a given final relative momentum corresponds to a small cone of final momenta for the incident particle. The magnitude of the uncertainty in the direction of $\mathbf{P}$ in the figure above is greatly exaggerated in order to render it visible.

R packet is just that of a free particle. Thus a specific direction for $\mathbf{p}$ leads to a narrow cone of $\mathbf{p}_i$ (the final momentum of the incident particle) values. However the distribution is generally so small that the cone of $\mathbf{p}_i$ values is in general very much smaller than the errors of measurement of direction. In this case we may with sufficient accuracy employ only $\mathbf{P} = \mathbf{P}_0$, the average value of the center of mass momentum. The transformation to the individual particle coordinates is then simply the usual transformation to laboratory coordinates and is the same transformation as that employed in the stationary state methods.

8. CONCLUSION

It has been shown that if the initial state of the collision system is such that the uncertainties in the momentum of the incident beam are sufficiently small then the cross section for the collision process is that given by Eq. 24 and that this result can be obtained from the stationary state eigenfunction (Eq. 25).

THE LOGICAL DESIGN OF AN IDEALIZED GENERAL-PURPOSE COMPUTER

BY

ARTHUR W. BURKS[1] AND IRVING M. COPI[1]

Part II[2]

5. ARITHMETIC UNIT

5.1. *Machine Arithmetic*

Before describing the operation of the *arithmetic unit* we must discuss the arithmetic of the machine.

A non-negative number which can be written in binary notation as $+ 0.x_1 \cdots x_{15}$ is **directly represented** in the machine by the word $0x_1 \cdots x_{15}$, and a negative number $- 0.x_1 \cdots x_{15}$ (< 0) by the direct representation of the positive number $2 - .x_1 \cdots x_{15}$. For example, $+.110 \cdots 0$ $(+ 3/4)$ is directly represented by $0.110 \cdots 0$, and $-.110 \cdots 0$ $(- 3/4)$ by $1.010 \cdots 0$. Clearly any number x in the range $-1 < x < 1$ can be directly represented in the machine to within 2^{-15}. Numbers outside this range can be indirectly represented in the machine through shifting them into the range by multiplying them by 2^{-n} with some appropriate n.

The binary point of a number is not directly represented in the machine. However, it is convenient to imagine, and sometimes to show, a binary point to the right of the leftmost digit of machine words.

It will be convenient to introduce an additional convention at this time. Just as a symbol like b_i may either name a wire or symbolize a function (which takes on one of the two values 0,1 at each time τ) (see Section 3), so a symbol like b may either name a sequence of sixteen wires $b_0, \cdots, b_{15}$ or symbolize a function (which takes on one of the 2^{16} values: $0.0 \cdots 00$, $0.0 \cdots 01$, $\cdots$, $1.1 \cdots 10$, $1.1 \cdots 11$ at each time τ). Our earlier use of t (Section 2) to stand for the trunk consisting of the wires $t_0, \cdots, t_{15}$ accords with this convention.

We shall introduce four operations on machine words: machine addition, machine subtraction, machine doubling, and machine halving; defining them so as to maintain a certain correspondence with the analogous operations of ordinary arithmetic. Where x and y are machine words which directly represent the numbers α and β, respectively, and z is the result of performing machine addition on x and y, then if $\alpha + \beta$ can be directly represented in the machine, it is directly represented by z, whereas if $\alpha + \beta$ cannot be directly represented in the machine, z directly represents a number congruent to $\alpha + \beta$ modulo 2. Similar

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remarks apply for the other three operations also. We will hereafter use the signs $x + y$, $-x + y$, $2y$, and $(1/2)y$ to denote our four machine operations. Each machine operation produces the same result as the corresponding arithmetic operation modulo 2, with a precision of one part in 2^{15}. For example, $0.110\cdots0 + 0.110\cdots0 = 1.100\cdots0$, that is, $3/4 + 3/4 = -(1/2)$, since $3/4 + 3/4 \equiv -(1/2)$ modulo 2; and halving $0.0\cdots01$ (2^{-15}) gives 0, while halving $1.1\cdots11$ (-2^{-15}) gives -2^{-15}.

The machine operations of addition, subtraction, doubling, and halving are the only machine arithmetic operations for which there are instructions in Fig. 2. However, other machine operations (for example, multiplication, division), as well as machine operations on numbers which cannot be directly represented in the machine, can be programmed; that is, there exist routines (sequences of instructions of Fig. 2) for making the machine perform these additional operations.

Our machine arithmetic operations are to be performed by nets constructed out of our logical elements, and these nets must all be realized by our *arithmetic unit*. The machine arithmetic operations performed on sixteen digit words (x,y) must be defined in terms of logical operations performed on the several bits (x_i, y_i) of those words.

We first define **machine addition.** Our task is to express

$$z = x + y \tag{A}$$

as a logical relation between x_i, y_i, and z_i. It is helpful to define the auxiliary word ξ whose i-1st bit ξ_{i-1} $(i > 0)$ is the ordinary carry digit from the addition of the three summands: x_i, y_i, and the carry digit ξ_i. It is clear that ξ_{15} is 0, and that z_i is 1 if and only if an odd number of the summands x_i, y_i, ξ_i are 1. The result concerning inequivalence established in Section 3 enables us to express this rule by

$$z_i \equiv (x_i \not\equiv y_i \not\equiv \xi_i). \tag{A$_1$}$$

And since ξ_{i-1} is 1 just in case at least two of the three summands are 1, we have the equations:

$$\xi_{i-1} \equiv (x_iy_i \vee x_i\xi_i \vee y_i\xi_i) \quad \text{for} \quad i > 0, \tag{A$_2$}$$

$$\xi_{15} = 0. \tag{A$_3$}$$

Before defining machine subtraction we introduce the machine operation of **complementation** C, whose field consists of words. We define $C(x)$ to be the machine representation of that number within the range $-1 < x < 1$ which is congruent modulo 2 to $2 - x$. The complementation operation is very useful because a negative number $-.x_1\cdots x_{15}$ is directly represented in our machine by $C(0.x_1\cdots x_{15})$. Machine com-

plementation is related to logical complementation (negation) in the following way. We define

$$\bar{x} = {}_{df}\bar{x}_0.\bar{x}_1\cdots\bar{x}_{15};$$

and refer to $\bar{x}$ as the **bitwise complement** of x. Now 2 is the arithmetic sum of $1.11\cdots11$ and 2^{-15}, and clearly $\bar{x} = 1.11\cdots11 - x$, whence

$$C(x) = \bar{x}_0.\bar{x}_1\cdots\bar{x}_{15} + 0.0\cdots01.$$

Machine complementation is therefore performed on a number by machine addition of 2^{-15} to the bitwise complement of that number.

Since the arithmetic operation of subtracting α from β is equivalent to the addition of the negative of α to β, we can define machine subtraction in terms of complementation and addition. Thus, the formula for **machine subtraction**

$$z = -x + y \tag{S}$$

becomes

$$z = C(x) + y = \bar{x} + y + 0.0\cdots01.$$

By using (A_1) and (A_2) we can translate $z = \bar{x} + y$ into logical terms; and since ξ_{15} is one of the summands in the rightmost digital position, the addition of 2^{-15} can be accomplished by setting ξ_{15} equal to 1. Thus, machine subtraction of the word x from the word y can be defined in terms of the following logical relations among the bits x_i, y_i, ξ_i, and z_i.

$$z_i \equiv (\bar{x}_i \not\equiv y_i \not\equiv \xi_i), \tag{S_1}$$

$$\xi_{i-1} \equiv (\bar{x}_i y_i \vee \bar{x}_i\xi_i \vee y_i\xi_i) \quad \text{for} \quad i > 0, \tag{S_2}$$

$$\xi_{15} = 1. \tag{S_3}$$

To define **machine doubling** we want to express

$$z = 2y \tag{D}$$

in logical terms. Since a 1 in the ith position has twice the value of a 1 in the $i+1$st position, doubling consists merely of shifting each digit of y left one binary position with respect to the binary point. We express (D) in terms of the logical operations on the bits y_i by

$$z_i \equiv y_{i+1} \quad \text{for} \quad i < 15, \tag{D_1}$$

$$z_{15} = 0. \tag{D_2}$$

Similarly, halving is accomplished by a right shift. We define **machine halving,**

$$z = (1/2)y, \tag{H}$$

by

$$z_i \equiv y_{i-1} \quad \text{for} \quad i > 0, \tag{H$_1$}$$

$$z_0 \equiv y_0. \tag{H$_2$}$$

5.2. Operation of the Control

Instructions 4, 5, 6, 7, 8, 9, 10, and 11 of Fig. 2 all involve the *arithmetic unit*. Before describing how the *arithmetic unit* is to function in their execution, we must explain briefly how the *control* affects the *arithmetic unit*. Exactly how the *control* performs the functions we are going to describe will be explained later in Section 6.

Since the instructions involving the *arithmetic unit* all refer to a "number in the *arithmetic unit*," it will be convenient to have a way of referring to this number. We will stipulate that it is located on the sequence of wires a.

At any time τ the wire sequence d_0, d_1, d_2, d_3 is in exactly one of sixteen distinct states. These sixteen states, together with the state of the *control clock*, completely determine the states of the thirteen control wires labeled A, S, and C with appropriate subscripts. (When the machine is inoperative, none of the wires is stimulated.) Figure 2 tells what control wires are activated for the various states of d_0, d_1, d_2, d_3 when the machine is operating; control wires not marked as active are stipulated to be inactive. The state 1111 can occur only by a mistake in programming; when it does occur, no control wires are activated, and it produces no effect.

When one of the instructions 4, 5, 6, 7 is being executed, the *control* will activate S_t and also send the address X to $s_4, \cdots, s_{15}$, thereby causing the *storage* to transmit the number in storage bin X to the trunk t. Hence we may assume that when these instructions are being executed the number in storage bin X is on t and therefore available to the *arithmetic unit*.

When instruction 10 or 11 is being executed, the *control* will activate S_{rt} or S_{rp}, respectively, and will also send the address X to $s_4, \cdots, s_{15}$, causing bin X to receive in the desired fashion any number on the trunk t. Hence we must so design the *arithmetic unit* that these instructions cause the contents of a to occupy t.

We may summarize the effect of the *control* on the *arithmetic unit* as follows. When one of the instructions 4 through 11 is being executed, control wires A_r, A_h, A_c, A_{1s}, A_{rs}, A_{11} are in the states shown in Fig. 2; at any other time these wires are all inactive. When instruction 4, 5, 6, or 7 is being executed, the number in bin X is also on the trunk t. Finally, when instruction 10 or 11 is being executed, the *control* causes whatever number the *arithmetic unit* places on t to be properly received by the correct bin.

5.3. The Functions of the Arithmetic Unit

In the previous subsection we described every possible way for information to be directed to the *arithmetic unit* by the *control*. In this subsection we consider three general functions performed by the *arithmetic unit* on the basis of this information, and explain how it performs the first two.

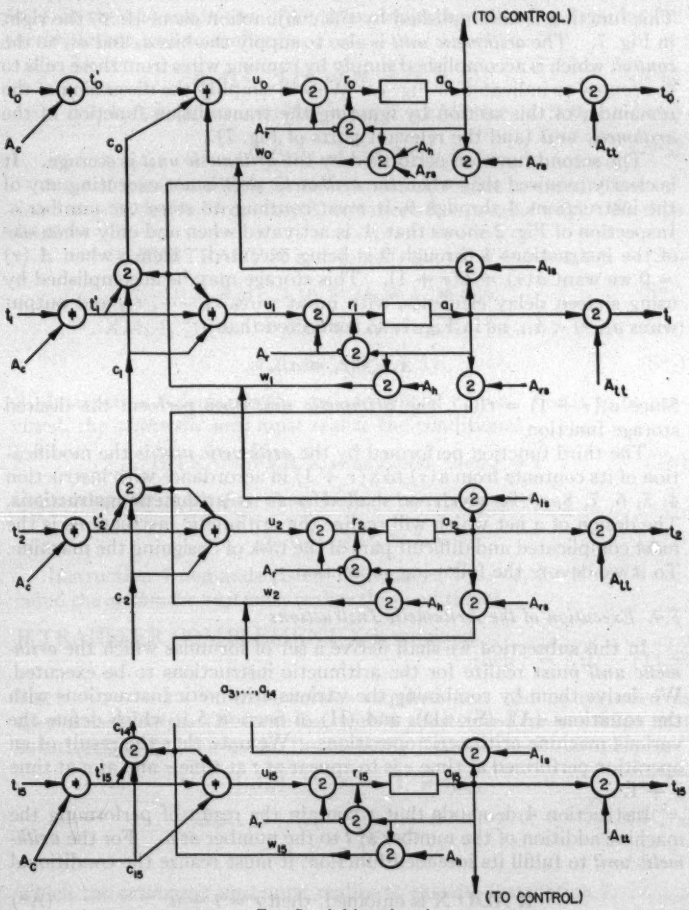

FIG. 7. Arithmetic unit.

The first function is the transmission of information. When either instruction 10 or 11 is executed, the *arithmetic unit* is to transmit its contents onto the trunk t. If either of these instructions is executed (see Fig. 2) A_{11} will be stimulated, hence we want the machine to realize the expression

$$A_{11} \supset (a_i \equiv t_i).$$

This function is accomplished by the conjunction elements to the right in Fig. 7. The *arithmetic unit* is also to supply the bits a_0 and a_{15} to the *control*, which is accomplished simply by running wires from these cells to the *control* as indicated in Fig. 7. We will simplify the discussion of the remainder of this section by ignoring the transmission function of the *arithmetic unit* (and the relevant parts of Fig. 7).

The second function performed by the *arithmetic unit* is storage. It is clearly required that when the *arithmetic unit* is not executing any of the instructions 4 through 9, it must continue to store the number a. Inspection of Fig. 2 shows that A_τ is activated when and only when one of the instructions 4 through 9 is being executed. Hence when $A_\tau(\tau) = 0$ we want $a(\tau) = a(\tau + 1)$. This storage may be accomplished by using sixteen delay elements, with input wires $r_0, \cdots, r_{15}$ and output wires $a_0, \cdots, a_{15}$, as in Fig. 7, so connected that

$$\bar{A}_\tau \supset (r_i \equiv a_i).$$

Since $a_i(\tau + 1) = r_i(\tau)$, the *arithmetic unit* does perform the desired storage function.

The third function performed by the *arithmetic unit* is the modification of its contents from $a(\tau)$ to $a(\tau + 1)$ in accordance with instruction 4, 5, 6, 7, 8, or 9, which we shall refer to as **arithmetic instructions.** The design of a net which will realize the arithmetic instructions is the most complicated and difficult part of the task of designing the machine. To it we devote the following subsection.

5.4. Execution of the Arithmetic Instructions

In this subsection we shall derive a set of formulas which the *arithmetic unit* must realize for the arithmetic instructions to be executed. We derive them by combining the various arithmetic instructions with the equations (A), (S), (D), and (H) of Section 5.1, which define the various machine arithmetic operations. We note that the result of an operation performed at time τ is to appear at r at time τ and at a at time $\tau + 1$.

Instruction 4 demands that r contain the result of performing the machine addition of the number at t to the number at a. For the *arithmetic unit* to fulfill its intended function, it must realize the conditional

$$\text{If ADD X is enjoined, then } r = t + a. \qquad (\text{A}^{\text{m}})$$

Now by Fig. 2, ADD X is the only instruction which involves the stimulation of A_r and A_h but not A_e. Hence the antecedent of (A^m) may be replaced by $A_rA_h\bar{A}_e$. And a translation of the consequent of (A^m) into a logical formula concerning the states of individual wires may be made by using (A_1), (A_2), and (A_3), substituting r_i for z_i, t_i for x_i, and a_i for y_i. In the *arithmetic unit* we are constructing, the auxiliary word ξ will be realized by the set of wires $c_0, c_1, \cdots, c_{18}$; hence we also substitute c_i for ξ_i in the (A) equations. We thus obtain the following formulas for the *arithmetic unit* to realize:

$$A_rA_h\bar{A}_e \supset \{r_i \equiv (t_i \not\equiv a_i \not\equiv c_i)\}, \qquad (A^m_1)$$

$$A_rA_h\bar{A}_e \supset \{c_{i-1} \equiv (t_ia_i \vee t_ic_i \vee a_ic_i)\} \quad \text{for} \quad i > 0, \qquad (A^m_2)$$

$$A_rA_h\bar{A}_e \supset \{c_{18} = 0\}. \qquad (A^m_3)$$

A set of formulas for the *arithmetic unit* to realize to execute instruction 5 may be derived from the (S) equations in a parallel way. We thus obtain

$$\text{If SUBTRACT X is enjoined, then } r = -t + a, \qquad (S^m)$$

$$A_rA_hA_e \supset \{r_i \equiv (t_i \not\equiv a_i \not\equiv c_i)\}, \qquad (S^m_1)$$

$$A_rA_hA_e \supset \{c_{i-1} \equiv (t_ia_i \vee t_ic_i \vee a_ic_i)\} \quad \text{for} \quad i > 0, \qquad (S^m_2)$$

$$A_rA_hA_e \supset \{c_{18} = 1\}. \qquad (S^m_3)$$

Instruction 6 demands the replacement of r by t. For it to be executed, the *arithmetic unit* must realize the conditional

$$\text{If TRANSFER X is enjoined, then } r = t. \qquad (T^m)$$

Consulting Fig. 2, we find that this can be represented as

$$A_r\bar{A}_h\bar{A}_e \supset (r_i \equiv t_i). \qquad (T^m_1)$$

Instruction 7 demands that r be replaced by $-t$; for it to be executed the *arithmetic unit* must realize the conditional

$$\text{If TRANSFER COMPLEMENT X is enjoined, then } r = C(t). \qquad (TC^m)$$

Since $-t = -t + 0$, the formulas required here can be regarded as special cases of the (S) formulas, with 0 substituted for y. After that substitution has been made, the resulting formulas easily simplify to

$$A_r\bar{A}_hA_e \supset \{r_i \equiv (t_i \not\equiv c_i)\}, \qquad (TC^m_1)$$

$$A_r\bar{A}_hA_e \supset \{c_{i-1} \equiv (t_ic_i)\} \quad \text{for} \quad i > 0, \qquad (TC^m_2)$$

$$A_r\bar{A}_hA_e \supset \{c_{18} = 1\}, \qquad (TC^m_3)$$

which the *arithmetic unit* must realize to execute instruction 7.

Instruction 8 demands that r be replaced by $2a$; for it to be executed the *arithmetic unit* must realize the conditional

$$\text{If DOUBLE is enjoined, then } r = 2a. \qquad (D^m)$$

Again substituting r for z and a for y, this time in the (D) formulas, and consulting Fig. 2, we obtain

$$A_r A_{1s} \supset \{r_i \equiv a_{i+1}\} \quad \text{for} \quad i > 0, \qquad (D^m_1)$$

$$A_r A_{1s} \supset \{r_{1s} = 0\}. \qquad (D^m_2)$$

These formulas must be realized by the *arithmetic unit* for it to execute instruction 8.

Instruction 9 demands that r be replaced by $(1/2)a$; for it to be executed the *arithmetic unit* must realize the conditional

$$\text{If HALVE is enjoined, then } r = (1/2)a. \qquad (H^m)$$

Again we substitute r for z and a for y, this time in the (H) formulas, and consult Fig. 2, to obtain

$$A_r A_{rs} \supset \{r_i \equiv a_{i-1}\} \quad \text{for} \quad i > 0, \qquad (H^m_1)$$

$$A_r A_{rs} \supset \{r_0 \equiv a_0\}. \qquad (H^m_2)$$

These must be realized by the *arithmetic unit* if it is to execute instruction 9.

We have now completed the task of deriving a set of formulas which the *arithmetic unit* must realize if it is to do its part in executing the arithmetic instructions. These are the fourteen formulas whose labels have superscript m and subscripts 1, 2, or 3. These fourteen formulas, together with the storage formulas (see Section 5.3)

$$\bar{A}_r \supset (r_i \equiv a_i) \qquad (K)$$

and

$$a_i(\tau + 1) \equiv r_i(\tau), \qquad (K')$$

provide a complete specification of the *arithmetic unit* (since we have agreed to neglect its transmission function). Any logical net which realizes these sixteen **defining formulas** will be a satisfactory *arithmetic unit*. It is readily verified that Fig. 7 is such a net, by observing that the last formula above is satisfied by the delay elements, and then taking each of the remaining formulas in turn and determining that under the conditions stated each r_i has the proper relations to a_i and t_i. Performing this verification will facilitate understanding how the *arithmetic unit* works.

6. CONTROL

6.1. *Address Counter*

We stipulated in Section 2 that apart from "jump" instructions (12, 13, 14 of Fig. 2) the computer executes in order the instructions stored in a sequence of bins of the parallel storage. To do so it must count off those instructions as they are executed. We need, then, an *address counter*, which can perform this function. One is diagrammed in Fig. 8 and operates as follows.

Each of its twelve cells a_4', a_5', $\cdots$, a_{15}' can receive and store one bit of information, so the *address counter* as a whole can receive and store a twelve digit number $x_4x_5\cdots x_{15}$. If all control wires are in state 0 and no keys are activated, each cell a_i' continues to store whatever information it contains. Any number in the *address counter* is the address of some bin of the parallel storage. To load the *address counter* with the address of the bin containing the first instruction we want the machine to execute, we must be able to change the contents of the *address counter*. The keys shown in Fig. 8 enable us to make any desired change. Activating the key at the lower left (when the machine is idle) clears all of its cells to 0, and activating the key of cell a_i' at time τ makes $a_i'(\tau + 1) = 1$; thus we can load the *address counter* with any number we please.

As remarked at the bottom of the *table of instructions* on Fig. 2, at every even numbered time $\tau = 2k$ when the machine is operating, control wires S_t and C_t are activated. At any such time, then, $C_t = 1$ permits the bit stored in cell a_i' to pass through the rightmost conjunction element onto s_i ($i = 4, 5, \cdots, 15$). Since $s_4, s_5, \cdots, s_{15}$ are input wires to the Address Decoder (see Fig. 5), and since S_t is also activated then, that storage bin whose address is contained in the *address counter* at any even numbered time $\tau = 2k$ will transmit its contents onto the trunk at that time.

Hence when the computer begins to solve a problem the first instruction it executes is the one stored in that bin whose address is stored in the *address counter*, usually bin 0. To keep track, whenever the *address counter* sends an instruction from the nth bin onto the trunk it must count, that is, add a 1 to the number it contains at that time.

(Any twelve digit number $x_4x_5\cdots x_{15}$ contained in the *address counter* is the address of some bin of the parallel storage, and can also constitute the address part of an instruction word, as in $d_0d_1d_2d_3x_4x_5\cdots x_{15}$. Whenever we consider the numerical aspect of a word we must keep in mind the discussion of range presented in Section 5, and the convention that a binary point is imagined to follow the leftmost bit of every number word. This convention should apply to instruction words too, for it is often convenient to perform arithmetic operations on instruction words. From this point of view the 4096 addresses of the 4096 bins of the parallel storage are 0.000000000000000, 0.000000000000001, $\cdots$, 0.000111111111111, and the number in the *address counter* is $0.000x_4x_5\cdots x_{15}$ or $(x_4x_5\cdots x_{15})2^{-15}$. Consequently the address

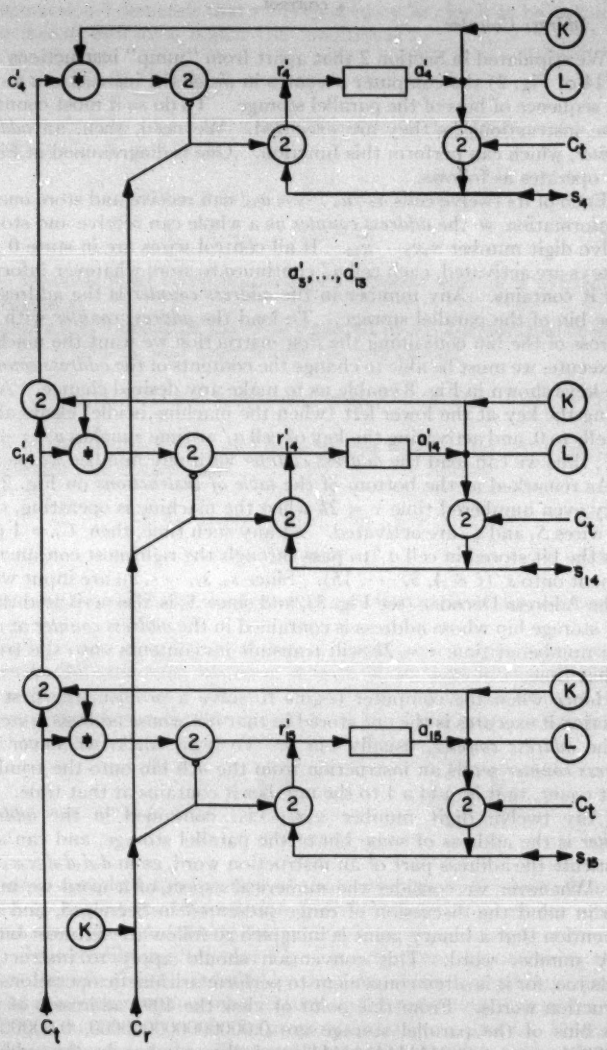

FIG. 8. Address counter.

of the next bin after that one is $(x_4x_5\cdots x_{15})2^{-15} + 2^{-15}$. It will, however, be convenient to continue to speak in this connection of the number in the *address counter* as $x_4x_5\cdots x_{15}$, and of its successor as that number + 1.)

Let us suppose that the number in the *address counter* at time τ is $x_4x_5\cdots x_{15}$, and to it we wish to add 1, which is $y_4y_5\cdots y_{15}$ where $y_{15} = 1$ and $y_4 = y_5 = \cdots = y_{14} = 0$. The sum $z_4z_5\cdots z_{15}$ of these two numbers is recursively defined (see Section 5) by the equations:

$$z_i \equiv (x_i \not\equiv y_i \not\equiv \xi_i)$$

and

$$\xi_{i-1} \equiv (x_iy_i \vee x_i\xi_i \vee y_i\xi_i) \quad \text{where} \quad \xi_{15} = 0.$$

Since $\xi_{15} = 0$ and $y_{15} = 1$, by the first defining equation we have $z_{15} \equiv \bar{x}_{15}$, and by the second defining equation we have $\xi_{14} \equiv x_{15}$. Now for every $i < 15$, $y_i = 0$, whence $\xi_{i-1} \equiv x_i\xi_i$, and also $z_i \equiv (x_i \not\equiv \xi_i)$. These formulas must be realized by the *address counter* if it is to perform its counting function correctly.

For the formulas developed in the preceding paragraph to be realized by a circuit, that circuit must contain wires corresponding to x_4, x_5, $\cdots$, x_{15}, to ξ_4, ξ_5, $\cdots$, ξ_{14}, and to z_4, z_5, $\cdots$, z_{15}, and these wires must be so connected that their behavior is described by those formulas. If net wires a_i', c_i', and r_i' correspond to x_i, ξ_i, and z_i, respectively, those wires must realize the equations $r_i' \equiv (a_i' \not\equiv c_i')$ and $c'_{i-1} \equiv a_i'c_i'$ for $i < 15$, $r_{15}' \equiv \bar{a}_{15}'$, and $c_{14}' \equiv a_{15}'$.

The diagram in Fig. 8 satisfies the preceding equations when $C_r = 0$ and $C_t = 1$. By assumption, $a' \equiv x$, and since $C_r = 0$ and $C_t = 1$ at every even numbered moment $\tau = 2k$, at every such moment r' will contain the immediate successor of the number in a, and that successor will occupy a at the following moment $\tau = 2k + 1$.

If at an odd numbered moment $\tau = 2k + 1$ a "jump" is to be made in executing instruction 12, 13, or 14 of Fig. 2, control wire $C_r = 1$ and $C_t = 0$, and the address X of bin X referred to in the "jump" instruction OBEY X (or OBEY X IF MINUS or OBEY X IF a_{15} IS 1) occupies wires s_4, s_5, $\cdots$, s_{15} (as will be explained in the following section). Since $C_t = 0$, no signal can pass from a_i' onto s_i through the cell's rightmost conjunction element; and since $C_r = 1$, no signal can pass from a_i' to r_i' through the conjunction element to the left of a_i'. But the lower left-hand conjunction element permits any bit of information on s_i to pass onto r_i' at time $\tau = 2k + 1$ and to occupy a_i' at time $\tau = 2k + 2$. Hence the address X of bin X referred to in the "jump" instruction OBEY X will be contained in the *address counter* at time $\tau = 2k + 2$. At that even numbered moment both S_t and C_t are automatically activated, which makes bin X of the parallel storage transmit its contents onto the trunk at time $\tau = 2k + 2$, and causes the Address Decoder to contain, at the following moment $\tau = 2k + 3$, the address of bin $X + 1$.

6.2. Control Clock

We wish to be able to start the machine at any time, and we want control wires S_t and C_t to be automatically activated at every even numbered moment while the machine is solving a problem. Our device for accomplishing this function is the *control clock*, which occupies the

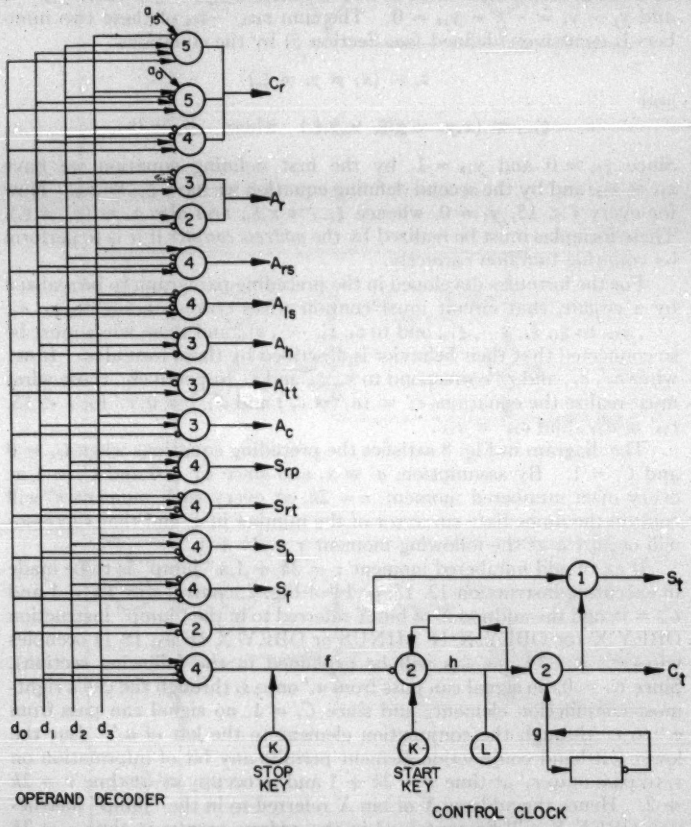

FIG. 9. Operand decoder and control clock.

lower right-hand part of Fig. 9. The behavior of the right-hand delay circuit is described by the equation $g(\tau) = (\tau \equiv 0 \bmod 2)$. Thus g is activated at $\tau = 0, 2, 4, \cdots, 2k, \cdots$ independently of anything that may happen elsewhere in the machine.

The left-hand delay circuit operates somewhat differently. If the wire f is inactive, then activating the *start key* at time τ will activate h. At time $\tau + 1$ the signal will emerge from the delay element above, and if f is still inactive, the signal will pass onto h again and up again to the delay element. Thus, so long as $f = 0$, activating the *start key* at time τ will activate h at times τ, $\tau + 1$, $\tau + 2$, $\cdots$.

The two delay circuits work together to produce the following result. So long as $f = 0$, if the *start key* is activated at either time $\tau = 2k - 1$ or $2k$, h will be activated at times $2k$, $2k + 1$, $2k + 2$, $\cdots$. The other input wire g is activated by the right-hand delay circuit at times $2k$, $2k + 2$, $2k + 4$, $\cdots$. Hence the output wire controlling C_t and S_t is activated at times $2k$, $2k + 2$, $2k + 4$, and so on. Once started, the computer's activity is cyclic, with control wires S_t and C_t activated at every even part of the cycle, starting at the moment the *start key* is activated if that is done at an even numbered moment, or at the following moment if the *start key* was activated at an odd numbered moment.

We wish also to be able to stop the machine, both manually and by instruction 1 of Fig. 2. To stop the machine we must activate f, which will prevent any signal from the upper delay element passing onto h, thus clearing the left-hand delay circuit. As Fig. 9 shows, f can be activated either by activating the *stop key* or by activating d_3 but not d_0, d_1, or d_2. Hence the operand 0001 signals the machine to stop. This circuit is part of the *operand decoder*, which is discussed in the following subsection.

6.3. Operand Decoder

We have shown in Sections 4 and 5 how the *storage* and the *arithmetic unit* function when their various control wires are activated, and we have listed in Fig. 2 the various different instruction words that activate different sets of control wires. It is the operand or first four binary digits of an instruction word which specifies which control wires are to be activated for the instruction to be executed. The *operand decoder* which occupies the left-hand part of Fig. 9 is a switching mechanism which activates the proper set of control wires when it receives the four digits of an operand.

The four wires d_0, d_1, d_2, d_3 are connected to various threshold elements in the way indicated. That the desired functional connections are realized by the *operand decoder* is easily verified. For example, the control wire A_τ, which must be activated for the execution of instruction 4, 5, 6, 7, 8, or 9 of Fig. 2 is, activated by any of their operands, as described by the following equation:

$$A_\tau \equiv (\bar{d}_0 d_1 \bar{d}_2 d_3 \vee d_0 \bar{d}_1 \bar{d}_2 d_3 \vee \bar{d}_0 d_1 d_2 \bar{d}_3 \vee d_0 \bar{d}_1 d_2 \bar{d}_3 \vee \bar{d}_0 d_1 \bar{d}_2 \bar{d}_3 \vee d_0 \bar{d}_1 \bar{d}_2 d_3).$$

That equation can be simplified to

$$A_r \equiv (\bar{d}_0 d_1 \vee d_0 \bar{d}_1 d_2),$$

which is obviously realized by the *operand decoder* as diagrammed in Fig. 9. The functional connections between operands and various sets of control wires could be specified in many different ways: the present arrangement was selected to permit simplifications of the kind indicated.

6.4. Operation of the Control

Four parts of our computer are represented by blocks in Fig. 1, and each has been explained in detail. We have now to explain the rest of the *control*, which consists of sixteen conjunction elements and sixteen delay elements, together with the wires connecting them to the other parts of the computer. Each component t_i of the trunk is connected to an input wire of one of these conjunction elements, whose other input wire is connected to control wire C_t. The output wires t_i'' of those conjunction elements lead to separate delay elements, whose output wires lead to d_0, d_1, d_2, d_3 and s_4, s_5, $\cdots$, s_{15}.

The functioning of these parts can best be explained by showing how they operate when the machine is executing an instruction under the direction of the *control*. When the machine is engaged in solving a problem, at any even numbered moment $\tau = 2k$ control wires S_t and C_t are automatically activated, making the *address counter* send the address of some storage bin down wires s_4, s_5, $\cdots$, s_{15} to the *storage*. The *storage* is thereby caused to transmit the instruction word from that bin onto the trunk. The sixteen digits of that instruction word pass through the sixteen conjunction elements above (since $C_t(2k) = 1$) to occupy the sequence of wires t''. At the following moment $\tau = 2k + 1$ (an odd cycle) the sixteen bits of the instruction word emerge from the sixteen delay elements. The first four bits (its operand) pass along d_0, d_1, d_2, d_3 to the *operand decoder*, and the last twelve bits (its address) pass along s_4, s_5, $\cdots$, s_{15} either up to the *address counter*, if its operand causes the *operand decoder* to activate control wire C_r, or down to the *storage* if its operand causes the *operand decoder* to stimulate either control wire S_t, S_{rt}, or S_{rp}.

Thus we see that at every even cycle the computer brings a new instruction word from *storage* onto its trunk, and at each following odd cycle the computer executes the instruction enjoined by that instruction word. We will now illustrate this process.

Consider the actual sequence of occurrences when the computer begins a routine whose first two instructions TRANSFER 101 and OBEY 6 IF a_{15} IS 1 are in bins 0 and 1, respectively. Now if the *address counter* contains all zeros, and we activate the *start key* at either time $\tau = 2k - 1$ or $\tau = 2k$, the machine will begin its run at $\tau = 2k$, when

$S_t = C_t = 1$, and the *address counter* transmits $00\cdots00$, the address of bin 0, along wires $s_4, s_5, \cdots, s_{15}$ to the *storage*. Bin 0 transmits its contents 0110000001100101 onto the trunk, up through the conjunction elements onto t_i''. Then at $\tau = 2k + 1$ those bits emerge from the delay elements, the operand 0110 going along d_0, d_1, d_2, d_3 to cause the *operand decoder* to activate control wires S_t and A_r. At the same time the address 000001100101 goes down $s_4, s_5, \cdots, s_{15}$ to the *storage*, which transmits the number x from bin 101 onto the trunk, and then into the cells of the *arithmetic unit*, since A_r is activated. At $\tau = 2k + 2$ we have $S_t = C_t = 1$ again, and the *address counter* transmits $00\cdots01$, the address of bin 1, along $s_4, s_5, \cdots, s_{15}$ to the *storage*, which transmits its contents 1110000000000110 onto the trunk, up through the conjunction elements onto t''. Then at $\tau = 2k + 3$ those bits emerge from the delay elements, 1110 along d_0, d_1, d_2, d_3 to the *operand decoder*, and $00\cdots0110$ onto $s_4, s_5, \cdots, s_{15}$. Now what control wires the *operand decoder* activates will depend upon the present contents of the *arithmetic unit*, which contains the number $x_0x_1\cdots x_{15}$. If the rightmost digit x_{15} of that number is 1, $a_{15} = 1$, and control wire C_r is stimulated by the *operand decoder*. In this case the address $00\cdots0110$ now occupying wires $s_4, s_5, \cdots, s_{15}$ enters the *address counter*, which at the following time $\tau = 2k + 4$ will transmit it down to the *storage* to cause the instruction word in bin 6 to pass onto the trunk. But if the rightmost digit x_{15} is 0, $a_{15} = 0$, and the *operand decoder* activates no control wires. In this case the address in the *address counter* remains $00\cdots010$, and at the following time $\tau = 2k + 4$ it is transmitted down to the *storage* to cause the instruction word in bin 2 to pass onto the trunk.

7. CONCLUSION

The logical design of an idealized general-purpose digital computer has now been completed. Our concluding remarks will concern the nature and utility of the procedure we have followed.

In Section 3 a rigorous diagrammatic symbolism was developed for describing computing circuits. This symbolism enjoys economy as well as precision and intuitive clarity, for at various stages we introduced simple diagrams as definitional abbreviations of complex nets to simplify our presentation (see particularly the last paragraph of Section 4.4). This diagrammatic language can be used to describe the logical structure of other machines, either idealized or actual. It would be of interest, for example, to work out the logical design of a Turing machine (**8**)[3] and express it in the symbolism here developed. Such machines are usually defined in terms of the instructions they execute (cf. our Fig. 2), and a description of how they could in principle execute these instructions would give additional insight into how they operate and perhaps be

[3] The boldface numbers in parentheses refer to the references appended to Part II of this paper.

suggestive with regard to the design of actual computers. It would also be of interest to have the logical design of a self-reproducing automaton (9), but for that our language would have to be enriched by the addition of at least one primitive symbol representing a device capable of grasping computer elements and interconnecting them.

In the foregoing we have treated our diagrams as symbols for expressing the structure of a computer. We have also remarked that the structure of a computer can be expressed rigorously, although less specifically, by means of the instructions it is to execute and the kinds of words it is to operate on, i.e., by means of the code language it is to "understand." The general problem of logical design can be regarded from the point of view of these instruction languages.

We can best show how this is done by reviewing our discussion of the *arithmetic unit*. That unit's functions were first specified by listing all instructions in whose execution it is involved (instructions 4 through 11 and 13 and 14 of Fig. 2), together with a precise explanation of the meanings of these instructions (Section 5.1) and a statement of what information the *arithmetic unit* will receive when a particular instruction is being executed (Sections 5.2 and 5.3). The problem of design is then the problem of translating the foregoing statement of what the *arithmetic unit* is to do into the symbolic logic of the formulae of Section 5.4 and thence into the diagrammatic language of Fig. 7. Thus we see that the process of design involves a double passage: first from a statement of the problem in the symbolism of ordinary language and arithmetic to its statement in terms of symbolic logic, and second from the latter into a statement in the diagrammatic language of the present paper.

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NUCLEAR ENERGY NEWS

ARGONNE LOW-POWER REACTOR

The Argonne National Laboratory, Lemont, Illinois, has been assigned responsibility by the Atomic Energy Commission for the design and development of a military nuclear reactor plant for production of electricity and for space heating. The Commission and the laboratory have selected the engineering firm, Pioneer Service and Engineering Company of Chicago, to work with the laboratory on the design of the reactor and the associated plant.

The project calls for the design of a low-power, heterogeneous, boiling reactor, to be known as the Argonne Low-Power Reactor (ALPR). The power plant that is proposed would produce a combined electrical and heat energy output of several hundred kilowatts. The reactor will consist of a pressure vessel containing an assemblage of enriched uranium fuel elements and a number of neutron-absorbing control rods submerged in water. The water will circulate through the reactor core by natural convection. Steam produced by the heat created by the fission of uranium atoms will be conducted directly to a turbine-generator.

The ALPR is planned as a prototype of nuclear plants for use in remote areas by the military services. The primary design objectives of the project are simplicity of the system, ease of transportability and construction, reliability, and ease of operation which would hold to a minimum the number of operating personnel required. A further objective is the design of equipment which can operate for extended periods of time without maintenance requirements.

PROPOSALS FOR SMALL-SCALE NUCLEAR POWER REACTORS

The Atomic Energy Commission announced in early February that it had received seven proposals to develop, design, construct and operate small capacity nuclear power plants in response to an invitation extended September 21, 1955, under the Commission's Power Demonstration Reactor Program. (See this JOURNAL, Vol. 260, p. 528.) That invitation was designed for development of small competitive nuclear power plants to benefit domestic users in certain areas, and to help meet the requirements of foreign countries as well.

Because the proposals are complex, they will require detailed analysis by the Atomic Energy Commission and, in several instances, clarification by the proposers will be necessary before estimates can be made of the dollar cost of services and materials requested of the

Commission. The estimates will be made available as soon as they can be completed. Each proposal will be evaluated to determine whether it constitutes an acceptable basis for negotiation of a contract for a fixed amount of assistance.

The proposals are:

1. Chugach Electric Association, Anchorage, Alaska, and Nuclear Development Corporation of America, White Plains, New York—sodium-cooled, heavy water moderated reactor; electrical capacity, 10,000 kilowatts; design agent, Nuclear Development Corporation; completion date, mid-1961.

2. City of Holyoke Gas and Electric Department, Holyoke, Massachusetts—gas-cooled reactor with closed-cycle gas turbine; electrical capacity, 15,000 kilowatts; design agent, Ford Instrument Company, Long Island City, New York; completion date, not determined.

3. City of Orlando, Florida—liquid metal fuel reactor; electrical capacity, 25,000 to 40,000 kilowatts; design agent and completion date, not determined.

4. City of Piqua, Ohio—organic moderated reactor; electrical capacity, 12,500 kilowatts; design agent, Atomics International, North American Aviation, Inc., Downey, California; completion date, 1960.

5. Rural Cooperative Power Association, Elk River, Minnesota—boiling water reactor; electrical capacity, 22,000 kilowatts; design agent, American Machine and Foundry, New York City; completion date, 1960.

6. University of Florida, Gainesville, Florida—pressurized light water reactor; electrical capacity, 2,000 kilowatts; design agent, not determined; completion date, 1959.

7. Wolverine Electric Cooperative, Hersey, Michigan—aqueous homogeneous reactor; electrical capacity, 10,000 kilowatts; design agent, Foster Wheeler Corporation, New York, New York, for reactor portion, and Worthington Corporation, Harrison, New Jersey, for secondary loop.

YANKEE NUCLEAR POWER PLANT

The revised proposal of the Yankee Atomic Electric Company to build a nuclear power plant under the Power Demonstration Reactor Program has been determined by the AEC to be an acceptable basis for negotiations. The original Yankee proposal was one of four submitted in response to the first invitation extended by the AEC under the Power Demonstration Reactor Program. (See this JOURNAL, Vol. 259, pp. 240, 531.)

The revised Yankee proposal is for a pressurized water reactor plant with a net electrical capability of 134,000 kilowatts. The Commission has determined that development of the proposed plant would advance the art of reactor technology by supplementing work

required in the development of a plant of the same general type being built at Shippingport, Pennsylvania, in a project jointly sponsored by the Commission and the Duquesne Light Company.

AEC REGULATIONS

During February the Atomic Energy Commission formalized a number of regulations that had been announced earlier in preliminary form.

Access Permits.—In the period from May 1955 to February 1956 the Commission issued more than 600 "Access Permits," under which business firms and individuals may gain access to Restricted Data relating to civilian uses of atomic energy. (See this JOURNAL, Vol. 260, p. 129.) Part 25 of Title 10 of the Code of Federal Regulations now formalizes the regulations that apply to the holders of these Access Permits and to later applicants for them. They are somewhat more liberal than the tentative ones that had been in effect previously. Inventions and discoveries resulting from access to Restricted Data will not need to be reported to the Commission. The specified categories of Secret Restricted Data, including a category on controlled thermonuclear processes, are listed as an aid to filing an application and also to facilitate the interchange of such data between companies holding Access Permits for the same categories of information. An Access Permit is subject to renewal at the end of two years, instead of only one as at first. The security regulations for the safeguarding of Restricted Data by holders of Access Permits are set forth in Part 95 of Code 10 of the Code of Federal Regulations.

Foreign Activity.—Part 110 of Title 10 of the Code of Federal Regulations authorizes, without advance Commission approval, unclassified foreign atomic energy activity by U. S. firms and individuals in countries not members of the Soviet bloc. This authorization, consistent with the U. S. national policy of encouraging the peaceful uses of atomic energy at home and abroad, is designed to help American companies and citizens doing business in foreign markets. Any who avail themselves of this privilege will be required to report to the Commission within 30 days of starting any activity which "constitutes directly or indirectly engaging in the production of any special nuclear material in any foreign country."

Licensing Reactors.—Two classes of licenses may be issued by the Commission under Part 50 of Title 10 of the Code of Federal Regulations for facilities such as nuclear reactors, isotope separation plants, and plants for processing irradiated materials containing special nuclear material. "Class 103" licenses are for commercial facilities and "Class 104" licenses are for reactors for use in medical therapy or in research and development. Licenses may be issued for periods up to 40 years. The regulation does not require a license for a plant

where nuclear fuels are fabricated into various shapes for insertion into reactors.

Patent Licenses.—Part 81 of Title 10 of the Code of Federal Regulations sets up procedures for the issuance by the Commission of licenses both for the royalty-free use of patents owned by the Commission and for royalties to be paid to private owners of patents related to atomic energy.

AEC Rules of Practise.—Part 2 of Title 10 of the Code of Federal Regulations formalizes the Commission's regulatory procedures. Its rules govern the conduct of proceedings before the Commission involving licenses and licensing.

USE OF AEC FACILITIES FOR PRIVATE WORK

The Atomic Energy Commission has established a policy under which organizations or individuals may be permitted use of Commission-owned facilities or equipment for their own purposes, in the interest of furthering private participation in the development of atomic energy for peaceful purposes. The Commission will permit the use of its facilities and equipment provided the user has appropriate security clearance, the work to be done will not interfere with Commission programs, and private facilities or equipment are not reasonably available. If necessary the Commission will establish priority for the use of the facilities and equipment.

Arrangements under which the facilities and equipment will be made available will include: (1) the assessment of charges based on either full recovery of costs of the work to the Commission or, where appropriate, on going commercial rates; (2) applicable patent provisions; (3) compliance with Commission regulations concerning health and safety, and security; (4) appropriate indemnification of the Commission from claims arising out of the performance of the private work.

As a minimum, the Commission will grant to the sponsor of privately-financed work involving use of AEC-owned facilities and equipment an irrevocable, non-exclusive, royalty-free license to use inventions or discoveries made in the course of this work. The granting of this license will be subject to the security restrictions and requirements of the Commission. Additional rights to such inventions or discoveries will be determined by the Commission.

RECOVERY OF URANIUM FROM FLUORIDE SCRAP

The Atomic Energy Commission has invited industry to participate in uranium processing through recovery of uranium contained in fluoride scrap generated at plants producing feed materials for use as reactor fuels. In addition to recovering uranium, it is expected that the program will also produce sizable quantities of fluorine for commercial use.

The fluoride scrap accumulates at production plants which convert uranium ores and concentrates into highly purified uranium compounds and metal, preparatory to their use in the production of fissionable materials. It is estimated that 4,000 to 8,000 tons of scrap materials will be available annually. Industrial participation will make possible the uranium recovery without expansion of government scrap processing facilities.

In the latter part of 1956, allowing time for industry to become familiar with processing technology involved, the Commission will invite bids on the purchase of the scrap material. Proposals will be requested on the basis of returning the recovered uranium to the Commission at pre-established prices and in a form acceptable for re-introduction into the Commission's production channels.

Firms interested in the program will be provided scrap material and specified technical data for making evaluation studies. The necessary license to possess the material and an Access Permit for the classified data which the Commission has developed on scrap recovery will be provided through the AEC's Division of Civilian Application.

AEC ISOTOPES PROGRAM

The AEC Isotopes Division has been placed under the Division of Civilian Application, and has been designated the Isotopes Extension. The change was effective January 1, 1956. The Isotopes Extension will remain at Oak Ridge, and licenses for procurement of radioactive isotopes will continue to be issued from there to distributors and users.

During 10 years of operation the Isotopes Division has issued nearly 50,000 authorizations (or licenses) for everyday uses of isotopes. These atomic energy by-products have been sent from AEC laboratories to all U. S. states and territories, and to 57 foreign countries.

In the first five years of the program almost all radioisotopes were produced in the Oak Ridge National Laboratory. Today, while Oak Ridge National Laboratory is the primary source of supply, other Commission laboratories also produce radioisotopes for an ever growing number of uses in medicine, agriculture, industry, and the physical sciences.

The transfer of the isotopes program to the Division of Civilian Application takes account of the fact that this Division, established June 29, 1955, has over-all responsibility for administering the Commission's licensing program designed to encourage development of a private atomic energy industry.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS

DICTIONARY OF COLOR NAMES

Thousands of color names are in use today among the various branches of art, science, and technology. Modern commerce and industry place great dependence on these names and use them to specify the colors of a large variety of products, from automobiles to women's clothing. Specific identification of color by name is also of great importance in zoology, botany, horticulture, geology, philately, and many other fields.

Over the years, each of these fields has gradually developed its characteristic color vocabulary. Some of these vocabularies are very similar—in fact they borrow from one another—while others are nearly or completely unintelligible to workers in another field. The color world has long needed a dictionary of color names that would correlate the color terms used in different fields and thus provide a common ground for scientists, businessmen, and the general public to compare and identify colors.

The National Bureau of Standards has recently compiled such a comprehensive dictionary with the cooperation of the Inter-Society Color Council (ISCC). The end result of four years of research and study, this dictionary not only includes color names from the various fields of application but also relates all the names listed to a common, fundamental system of designation. It lists 7500 individual color names and defines them in simple, accurate terms that can be easily understood by persons working in different fields. Included are such prosaic color names as *red*, *pea green*, and *dull reddish yellow* along with more fanciful names such as *kitten's ear*, *vamp*, and *French nude*.

Listings are arranged in such a way as to permit anyone to translate rapidly and accurately from one color vocabulary to another. For example, the dictionary shows that *griseo-viridis* (from biology) is equivalent to *serpentine* (fashion), to *mint green* (mass market), and finally to *light green* (ISCC-NBS color designation). The dictionary has been published as NBS Circular 553,[1] *The ISCC-NBS Method of Designating Colors and a Dictionary of Color Names*, by Kenneth L. Kelley and Deane B. Judd.

Earlier dictionaries of color names, which define each color by an actual color swatch, have had limited usefulness in industry because they fail to show the extent of the color range indicated by the name. In the Bureau's color dictionary this defect has been overcome by the use of a method of designating colors through combinations of 21

[1] Available from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., $2.00 ($2.50 foreign).

generic terms—red, yellow, green, light, dark, and so on. In this system all possible colors are divided into 267 groups, each identified by combinations—such as *dark red* or *light green*—of the 21 generic terms. These definitions provide an easily understood, concise, systematic color language composed of simple unambiguous hue names and modifiers.

The terms used to designate colors represent a refinement of a method outlined originally by the Inter-Society Color Council and developed by the National Bureau of Standards in 1939.[2] These color designations are precisely defined in terms of the standards of the Munsell color system, and the designation for any color may be read quickly and conveniently from color-name charts. It is now possible to determine the ISCC-NBS color designation for a sample whose color is specified in terms of another system of colored samples if the Munsell notations of the reference samples are known.

Circular 553 is made up of four parts. The first part describes the techniques for determining the ISCC-NBS color designations for the colors of different types of samples. Next come the color-name charts from which the designations are obtained. The third part lists all of the color names studied in groups that are synonymous or nearly so; there are 267 groups, one for each of the ISCC-NBS color designations. The last part, which might be called the dictionary proper, is an alphabetical list of all of these color names, together with their designations in the ISCC-NBS system. With each color name is given a code letter indicative of the field in which it is used. After each ISCC-NBS designation defining a color name is a number referring to the section in the third part where all synonymous color names will be found.

All double color names like *turquoise blue* and synonyms within one system are cross-referenced. For instance, under "blue" are five pages of cross-referenced blues, from "blue, abstract" to "blue, zephyr." Altogether, there are about 17,000 entries, with more than 600 entries under blue alone.

The dictionary of color names should prove very useful to anyone who is looking for a name to describe a certain color. If he wishes to find color names for use in a particular field, such as dyes, philately, or biology, he has only to look for the key letter denoting that field among the entries in the dictionary. On the other hand, if he has already chosen a color name, he can use the same code letters to determine in what field it has been used before. The manner in which the dictionary is compiled makes differences in meanings of the same color name in different fields immediately evident. Industrial designers and executives who wish to coin new color names should also find this dictionary useful not only as a source of suggestions but also to make sure that the new name does not already have an established meaning.

[2] "Method of Designating Colors," by Deane B. Judd and Kenneth L. Kelly, *J. Research NBS*, Vol. 23, p. 355 (1939) RP 1239.

NBS-AF PANORAMIC X-RAY MACHINE

Research at the National Bureau of Standards has produced an X-ray machine that rapidly takes a single panoramic X-ray picture of the entire dental arch. Developed by the Bureau in cooperation with the U. S. Air Force Dental Service and the USAF School of Aviation Medicine, the panoramic X-ray machine should be particularly useful to the armed forces in making full-mouth dental X-ray surveys of inductees on entering and leaving the service. It will save much of the time required by present techniques, in which up to 14 small films are exposed separately for a full-mouth survey. The device should find application wherever large numbers of people are examined for dental defects, and its principles can be applied to radiography of other parts of the body.

In the NBS-AF panoramic machine, the film is placed outside the patient's mouth and is exposed by passing a narrow beam of X-rays through his head from the rear. A panoramic X-ray picture of all the teeth and associated structures is obtained on a single 5 × 10-in. film in about 40 secs. Thus the problem of handling many small film packets during exposure and processing is eliminated. At the same time more comprehensive radiographs are produced, giving more general diagnostic information than do conventional full-mouth X-ray surveys.

For several years investigators in both the United States[1, 2] and Finland[3] have been seeking to develop a rapid, practicable method for making panoramic X-rays of the entire dental arch. However, the procedures that have been tried have been rather cumbersome—they required rotation of the patient or fitting films inside the mouth. Because of the large number of full-mouth surveys that must be made of military personnel, particularly at induction stations, a rapid, dependable method was needed to replace conventional radiographic techniques. Funds for research in this field were therefore provided by the Air Force. As a result, a more flexible and simple panoramic machine was developed by Col. D. C. Hudson, NBS guest worker from the U. S. Air Force Dental Service, and J. W. Kumpula of the Bureau staff, with the cooperation of members of the NBS electronic instrumentation laboratory.

In this machine, an X-ray source and film holding device follow semicircular paths on opposite sides of the patient's head. The film holder travels in front of the patient, the X-ray source behind him. Movement of source and film is so coordinated that only those structures of the dental arch desired in the finished film are sharply projected while other overlying structures are not.

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[1] U. S. Patent 2, 476,776, issued to H. Smathers.

[2] "Panographic Radiography," by R. J. Nelsen and J. W. Kumpula, *J. Dent. Res.*, Vol. 31, p. 158 (April 1952).

[3] "Pantomography in Theory and Use," by Y. V. Paatero, *Acta Radiologica*, Vol. 41, p. 321 (1954).

The X-ray source and film holder are suspended from opposite ends of a horizontal arm that rotates about a central vertical axis. A narrow beam of X-rays emerges from a slit in the exit cone of the X-ray source, passes through the subject's head, and enters a corresponding slit in the film holder just beyond his teeth. Meanwhile, the film, in a carrier within the holder, travels horizontally in a direction opposite to that of the holder and at such a rate that an X-ray shadow of each successive tooth falls on successive areas of the film.

To avoid unwanted X-ray shadows from overlying bony structures, the X-ray beam is made to enter the patient's head largely through the soft tissue between the vertebral column and the bone of the jaw—first from one side, then from the other. As the system rotates, the axis of rotation of the X-ray beam is in this soft tissue, which is much more transparent to X-rays than are the harder structures. Thus, since the X-ray beam is in constant motion, shadows cast by intervening bone and other tissue between the point of entry and the dental arch move across the film too rapidly to obscure detail in the exposure. As a result, clearer lateral jaw radiographs can be obtained than with conventional techniques.

A simple mechanical system automatically varies the rate of film travel to conform to the size and shape of the human dental arch. This device consists of a cable wound about a cam which is curved in the shape of an average dental arch. The free ends of the cable are connected through pulleys to the film carrier. As the horizontal arm rotates at constant speed, the film moves at a rate determined by the curvature of the cam surface at the point where the cable is leaving the cam. The cam is fixed at the axis of rotation, while an electric motor rotates the arm supporting the X-ray source and film holder. A central panel controls the motor as well as X-ray voltage and current.

The first model of the panoramic X-ray machine is now undergoing performance studies. In this model the patient sits in a dental chair beneath the rotating arm and is positioned by means of a chin support pivoted from a stationary point on the machine. The chin support serves to steady the head and to place the dental structures in the proper position for best projection onto the moving film.

With the cooperation of the Naval Medical Research Institute, a study[4] was made of the radiation levels produced by the panoramic X-ray machine at points in and about a phantom head constructed of tissue-equivalent wax molded upon an adult human skull. By the use of small ionization chambers—about 30 mm^3 in volume—it was found that the panoramic device produces lower radiation levels than the conventional 14-film intraoral technique produces at corresponding points. The reduction in radiation received by the patient is due to the small area covered by the beam of X-rays during panoramic exposure and the fact that no overlapping occurs.

[4] "Ionization Chambers for Radiation Data During Dental X-Ray Exposure," by D. C. Hudson and J. W. Kumpula, *Armed Forces Medical Journal*, Vol. 6, p. 1131 (August 1955).

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

March 21, 1956

The Stated Monthly Meeting of the members of The Franklin Institute was held on March 21, 1956 in the Lecture Hall. The President, Mr. S. Wyman Rolph, called the meeting to order at 8:15 P.M. The Secretary, Mr. H. V. Bail, acted as Secretary of the meeting. Approximately 135 members and guests were in attendance.

The President stated that the minutes of the February Stated Meeting had been printed in the JOURNAL for March, which had already been distributed and asked if there were any corrections or changes in them. There being none, he ordered them approved as submitted.

The President then welcomed the officers and members of the Philadelphia Society for Promoting Agriculture to their sixteenth joint meeting with The Institute.

He announced that he had been delegated to bring to the Society a token of deep appreciation from the Congress of the United States—one of the gold medals voted by Congress to the Scientific Societies and Institutions of which Benjamin Franklin was a member. In presenting the medal, the President stated that he had been authorized to say that it was being awarded not only in memory of Benjamin Franklin, but also in recognition of the many wise and good men who had been and were members of the Society.

After presentation of the Medal to Mr. J. Gordon Fetterman, President of the Society, the President called upon him to take the chair and carry on the meeting. Mr. Fetterman graciously accepted the Medal and expressed his appreciation to Congress for its award to the Society. He then introduced Dr. William L. Henning, Secretary of Agriculture for Pennsylvania and presented him with the Society's annual Medal for outstanding work in the Science of Agriculture.

Dr. Henning then delivered an address on the subject, "The Agricultural Situation Today," in which he outlined the state of Agriculture in the Nation and specifically in Pennsylvania, which was received with much satisfaction by the audience.

After a question period the meeting adjourned at 9:50 P.M. with a rising vote of thanks to Dr. Henning.

H. V. BAIL
Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, March 14, 1956.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MARCH 14, 1956.

MR. FRANCIS B. FOLEY in the Chair.

The following report was presented for final action:

No. 3225: One-step Photographic Process.

This report recommended the award of a Howard N. Potts Medal to Edwin H. Land, of Cambridge, Massachusetts, "In consideration of his ingenious development of a practical hand camera and a process to expose and to develop the negative and create a good positive simultaneously, all within approximately a minute."

JOHN FRAZER,
Secretary to Committee

MEMBERSHIP

ACTIVE MEMBERS ELECTED BY THE BOARD OF MANAGERS

March 21, 1956

ACTIVE FAMILY

Herman A. Affel, Jr.
J. Sellers Bancroft
George Burnham, IV
Arthur R. Cannon
Herbert Fairfax Leary
John A. Mayer
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FINK, DONALD G.: Danger! The Radio Spectrum is Bursting at the Seams
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HEYDA, JAMES F.: An Elementary Derivation of the Formula for the Windage Jump of a Spinning Shell
MITCHELL, E. H.: Analysis of Magnetic Circuits with Given D.C. Excitation

NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

PICTORIAL LIFE OF FRANKLIN

The Franklin Memorial Hall has been further embellished with a pictorial life of Benjamin Franklin. Because of space limitations and the architectural features of the Hall, it was decided to encompass the life in four panels, to be displayed in especially designed and illuminated cases. Four principal aspects of the life were chosen as most representative for illustration—printer, citizen, scientist, and statesman—and, while these overlap to some extent, they embrace practically the whole span of an active and varied life.

Within the arbitrary limits of these four panels all the significant events of Franklin's life have been noted and, where it was found possible, appropriate pictures have been chosen to illustrate events, portray people, or picture localities. While it might have been possible to use modern photographs to show some of the places with which Franklin was associated, it was decided to employ only old prints that would show these scenes as they appeared in Franklin's day. This had both advantages and disadvantages.

It was, for instance, a decided advantage to select a picture of the Charterhouse Square to illustrate the London of Franklin's first visit. For, while he had no particular association with this Square, the picture shows in the right background the tower of St. Bartholomew's Church and the dome of St. Paul's. Thus, the visitor is able to locate two significant places associated with Franklin at this time. Samuel Palmer's printing house, where he was first employed, faced St. Bartholomew's, and Little Britain, where he had lodgings, is a crooked thoroughfare running from the church to St. Paul's.

A possible disadvantage may be perceived in the use of the old print to illustrate the famous kite experiment, with its rather amusing implications to the modern viewer, but the selection is justified on the grounds that this is how the men and women of Franklin's day visualized what had taken place on that occasion. No explanation can be offered for the significance of the dog and the rooster in this contemporary print.

The picture of the interior of Congress Hall may arouse some comment, but it should be pointed out that this not the arrangement with which we have been so familiar for many years, but shows the room as it was toward the end of the eighteenth century.

Another picture of unusual interest is that of Franklin as a fireman. This was painted five years after the great man's death for the Union Fire Company, which he had founded. An interesting detail in this portrait is the picture of the fire company's headquarters on Grindstone Alley (now a part of Church Street), and is the only known picture of the building.

To enable the visitor to recall the progress in thought and action that parallelled Franklin's career, portraits of prominent contemporaries in several fields are included in each panel.

Finally, to complete the picture, small objects associated with Franklin are displayed in each appropriate case. This permits a better view of the Institute's collection of Frankliniana. Material too large for inclusion is shown in other display cases ranged around the Hall. An interesting temporary addition to this material has recently been added through the courtesy of Miss Katherine N. Bradford, of Philadelphia. This consists of a handsome silver inkstand and a pair of candlesticks, all of which graced the desk of Franklin during his residence in Craven Street, London. The inkstand was bequeathed by him to Polly Stevenson, his landlady's daughter to whom some of his most charming letters were addressed. Apart from its historical association, the inkstand is a beautiful example of the silversmith's delicate art.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, M.D., FOUNDER
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HUMAN BLOOD PLASMA PRESERVED FOR SIX YEARS AT ROOM TEMPERATURE. PHYSICO-CHEMICAL OBSERVATIONS

BY

LAURA E. KREJCI, LUCILE SWEENY AND JOHN HAMBLETON

During World War II the preservation of human blood plasma in the liquid state at room temperature was carefully investigated by the Naval Medical Research Institute (1, 2, 3, 4, 5). As a result of this investigation, it was established that plasma prepared by the closed vacuum technique, with scrupulous bacteriological control, can be preserved for at least three years and used for transfusion "with safety and benefit." The loss of labile constituents such as complement or the blood coagulation factors, which invalidates its use in infections or hemorrhagic diatheses, is counterbalanced by the striking decrease of untoward reactions after storage for six months or longer; the anti-shock properties remain unaltered. More recently, storage at room temperature for six months has been found to afford protection also against transmission of viral hepatitis (6, 7, 8). The superiority of stored liquid plasma for clinical conditions in which the labile components are not specifically required, prompts a report of physico-chemical measurements on liquid plasma preserved six years at room temperature. These measurements are of interest because of possible implications with regard to the conditions of storage.

During the course of its investigation, the Naval Medical Research Institute placed at our disposal four representative samples of plasma which had been preserved for three years at room temperature; these, in common with all plasmas prepared under this program, contained glucose at a concentration of 5 per cent and merthiolate at a dilution of 1:10,000. As controls, two samples of fresh plasma were supplied which had been prepared in the same way as the preserved samples, except that one contained no added glucose or merthiolate. On examination in the Tiselius electrophoresis apparatus, in a phosphate saline buffer solution of pH 7.8, ionic strength 0.2, it was found that preservation for three years had brought about a large increase in the component with the mobility of alpha globulin at the expense of all the gamma globulin, all the fibrinogen and part of the beta globulin; the mobilities were 20–25 per cent greater than in fresh plasma, and the boundaries were broadened (9).

Six years later, the only one of the six plasmas that had escaped

contamination during previous samplings was re-examined by electrophoresis; this was the control plasma (A) which had been supplied fresh without glucose or merthiolate. Figure 1*b* shows the pattern obtained with the phosphate saline medium used for the earlier experiments, and Fig. 2*b* the pattern in sodium veronal buffer solution of pH 8.6, ionic strength 0.1. For comparison, Fig. 1*a* shows the pattern for

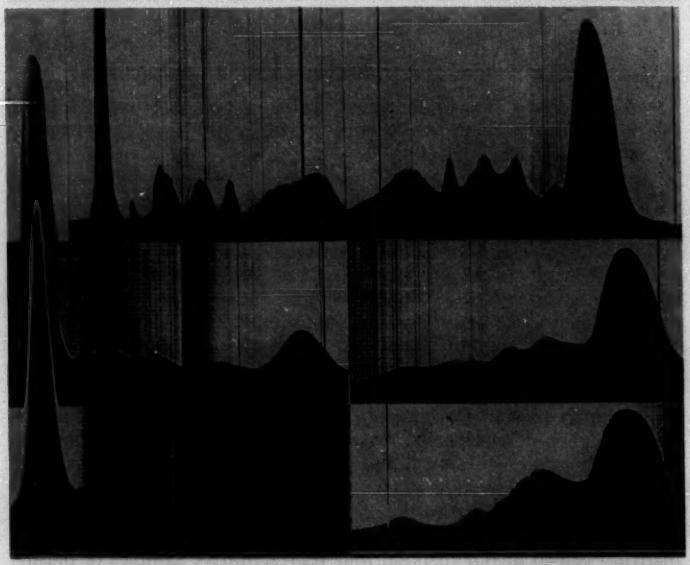

FIG. 1. Electrophoretic patterns of plasma in buffer solution of pH 7.8 containing 0.02 M sodium phosphates and 0.15 M sodium chloride. Ascending boundaries to the left, descending boundaries to the right. Migration for 5 hours at 6 V/cm. Plasma diluted 1→2.

(*a*) (*Top*) Plasma A, fresh.

(*b*) (*Center*) Plasma A, stored six years at room temperature without glucose.

(*c*) (*Bottom*) Plasma 291-4, stored three years at room temperature with 5 per cent glucose.

the same plasma (A) when fresh, and Fig. 1*c* that for one of the plasma pools (291-4) (9) after three years of storage with 5 per cent glucose. No measurements in sodium veronal buffer solution were made on Plasma A when fresh; Fig. 2*a* is therefore a fresh normal plasma selected for comparison.

Except for the dissymmetry of the albumin boundaries in the phosphate saline medium, the patterns obtained with the two buffer solutions were similar. In sodium veronal buffer solution the mobilities were normal, and the pattern differed from that of fresh plasma in the

poorer resolution of the boundaries, in the absence of fibrinogen, and in the lower proportion of the slower globulins. On the other hand, in the phosphate saline buffer solution the mobilities, like those of the three-year-old samples, were greater than in fresh plasma. The pattern, however, differed from that of the fresh plasma less than did those of the three-year-old glucose-containing plasmas studied six years earlier.

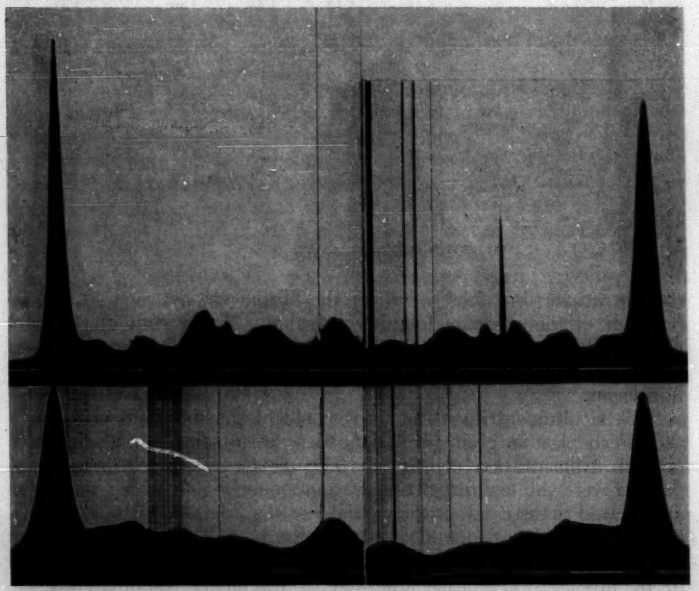

FIG. 2. Electrophoretic patterns of plasma in sodium veronal buffer solution of pH 8.6, ionic strength 0.1. Ascending boundaries to the left, descending boundaries to the right. Migration for 2-1/2 hours at 9 V/cm.

(*a*) (*Top*) Fresh normal plasma, diluted 1→5.

(*b*) (*Bottom*) Plasma A, stored six years at room temperature without glucose; diluted 1→4.

The higher mobilities of the six-year-old plasma in phosphate saline buffer solution proved to be almost identical with the normal, unchanged mobilities in the veronal buffer solution (Table I). This made it possible to identify the plasma constituents in spite of their increased mobilities and to determine what changes had occurred in the plasma during its six years of storage. As shown in Table II, about 0.8 g. of protein per 100 ml. of plasma, or 13 per cent, was lost. There was no significant change in the albumin and alpha globulin

TABLE I.—*Electrophoretic Mobilities of Fresh and Stored Plasmas.*

		Mobilities (cm. × 10^5/sec. volt/cm.) in veronal buffer solution of pH 8.6, ionic strength 0.1.					
		Albumin	Globulins				
			Alpha$_1$	Alpha$_2$	Beta	Fibrinogen	Gamma
Fresh Normal Human Plasma	Desc.	6.27	5.29	4.37	3.20	2.56	1.77
	Asc.	6.17	5.33	4.46	3.21	2.64	1.70
	Average	6.22	5.31	4.41	3.20	2.60	1.74
Plasma A, Stored Six Years Without Glucose	Desc.	6.06	5.07	4.40	3.51	2.55	
	Asc.	6.16	5.12	4.45	3.46	2.37	
	Average	6.11	5.10	4.43	3.48	2.46	
		Mobilities (cm. × 10^5/sec. volt/cm.) in buffer solution of pH 7.8 containing 0.02 M sodium phosphates and 0.15 M sodium chloride.					
Plasma A, Fresh	Desc.	4.8	4.0	3.2	2.5	1.8	1.0
	Asc.	4.9	4.2	3.7	2.8	2.1	1.1
	Average	4.85	4.1	3.45	2.65	1.95	1.05
Plasma A, Stored Six Years Without Glucose	Desc.	6.25	5.05	4.35	3.59	2.65	2.37
	Asc.	5.95	4.86	4.58	3.63		2.38
	Average	6.10	4.95	4.46	3.61	2.65	2.38
Plasma 292-4, Stored Three Years with 5 Per Cent Glucose	Desc.	6.1		4.0	2.8		
	Asc.	6.1		4.6	2.7		
	Average	6.1		4.3	2.75		

concentrations; but most or all of the fibrinogen, between 40 and 50 per cent of the gamma globulin, and about 20 per cent of the beta globulin had vanished. The remaining gamma globulin appeared to consist largely of gamma$_1$ globulin, which migrates close to the fibrinogen.

In the ultracentrifuge the six-year-old plasma behaved very much as a fresh plasma, giving well-defined sedimentation boundaries with normal sedimentation constants. The most obvious features of the pattern were the low proportion of component $s = 7$, which was to be anticipated as the result of the partial loss of gamma and beta globulins, and the somewhat poorer definition of the boundaries for components $s = 7$ and $s = 18$. Apparently the proteins shown by electrophoresis to be missing were either digested to fragments small enough for removal by dialysis, or were aggregated to particles large enough to be removed from the solution during acceleration of the ultracentrifuge.

TABLE II.—*Changes Produced in Plasma A (Glucose-Free) by Six Years of Storage.*

	Composition: Grams Protein/100 ml. of Plasma						
	Albumin	Globulins					Total
		Alpha$_1$	Alpha$_2$	Beta	Fibrinogen	Gamma	
Phosphate Saline Buffer							
Plasma A, Fresh	3.66	0.15	0.62	0.65	0.35	0.87	6.30
Plasma A, Six Years Old	3.63	0.25	0.60	0.51	0.00	0.49	5.48
Difference	−0.03	+0.10	−0.02	−0.14	−0.35	−0.38	−0.82
Veronal Buffer Solution:							
Plasma A, Six Years Old	3.30	0.51	0.71	0.55	0.00	0.46	5.53

Without knowledge of the composition of the glucose-containing plasmas when fresh, no quantitative determination can be made of the changes produced in them by three years of storage. The non-protein nitrogen liberated was equivalent to the hydrolysis of 3–4 per cent of the total protein (5). This is only about one-third as much protein as was lost during six years of storage without glucose; but since all the glucose-containing plasmas supplied for transfusion were accompanied by instructions (1) emphasizing the tendency of the plasma to form a precipitate on standing, and the need for a filter in the intravenous set, it seems reasonable that some protein may have been lost by precipitation as well.

The proteins resisting hydrolysis or precipitation appear to have been more nearly normal after six years of storage without glucose than after three years of storage with glucose. During this period end-groups were apparently formed or liberated which were capable of binding phosphate ions, with consequent increase of mobility. These groups appeared not to bind veronal ions, however, and not to alter the net charge on the proteins at pH 8.6. In the presence of glucose further changes must have occurred during storage. The resolution of the globulin boundaries in the three-year-old glucose-containing plasma was even poorer than in the six-year-old glucose-free plasma; and the high proportion of material migrating with the mobility of alpha globulin led to the suggestion that this might represent an aggregation product formed at the expense of the gamma globulin, the fibrinogen, and part of the beta globulin.

In 1951 and 1952 Dr. Lloyd R. Newhouser (8) obtained electrophoresis patterns on liquid plasma prepared in 1940, 1941, 1942, and 1943 with 5 per cent added glucose and 1:10,000 merthiolate; these patterns were all similar to those for the three-year-old plasmas. The striking uniformity of liquid plasmas stored from three to twelve years with glucose strengthens the validity of our observations on liquid plasma stored six years without glucose. It appears that the preservation of liquid plasma for clinical use may not be promoted by the addition of glucose but may possibly be more successful without it.

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THE PRINCIPLES OF CHEMICAL EQUILIBRIUM, by K. G. Denbigh. 491 pages, diagrams, 5¾ ×8¾ in. New York, Cambridge University Press, 1955. Price, $7.50.

This is an English textbook of chemical thermodynamics and a pretty good one, too. It is actually more than a textbook of thermodynamics in that it covers most of the fundamentals of physical chemistry. There are three sections: (1) The Principles of Thermodynamics, (2) Reaction and Phase Equilibria, which deals with the applications of thermodynamics to gases, solutions, phase equilibria, and reaction equilibria, and (3) Thermodynamics in Relation to the Existence of Molecules, which includes an introduction to statistical mechanics with applications to gases, the third law, adsorption and solutions. There is a final chapter called Chemical Equilibrium in Relation to Chemical Kinetics, which covers the basic applications of thermodynamics to kinetics.

Commendable features of the book are the logical and lucid explanations of difficult points, the many practical examples in the text, and the many problems at the end of the chapters with answers and explanations in the appendix.

The notation is slightly different from the Lewis and Randall notation which has practically become the standard in America. This is hardly a criticism, however, since the student must learn all the different ways the symbols are used.

The text would be most suited to the graduate student in physical chemistry. In fact, a two-semester course in advanced physical chemistry could be built around it. It would not be suitable for a first course in physical chemistry. The practicing chemist will find it very useful for review and reference.

PHILIP HANST

SOLID STATE PHYSICS, Vol. I, Advances in Research and Applications, edited by F. Seitz and D. Turnbull. 469 pages, diagrams, 6 × 9 in. New York, Academic Press Inc., 1955. Price, $10.00.

This volume is the first of a projected series in which significant developments in solid state physics will be presented by means of comprehensive and authoritative reviews by qualified scientists. The editors plan to include three types of articles:

"1. Broad elementary surveys that have particular value in orienting the advanced graduate student or an investigator having little previous knowledge of the subject;

"2. Broad surveys of fields of advanced research that serve to inform and stimulate the more experienced investigators; and,

"3. More specialized articles describing important new techniques, both experimental and theoretical."

In view of the recent tremendous increase of information about the solid state, it is obvious that this series will be of considerable value to anyone interested in solid state research.

The first volume includes the following articles:

1. Methods of the One-Electron Theory of Solids, J. R. Reitz
2. Qualitative Analysis of the Cohesion in Metals, E. P. Wigner and F. Seitz
3. The Quantum Defect Method, F. S. Ham
4. The Theory of Order-Disorder Transitions in Alloys, T. Muto and Y. Takagi
5. Valence Semiconductors, Germanium and Silicon, H. Y. Fan
6. Electron Interaction in Metals, D. Pines

Complete subject and author indexes have been provided.

The articles have been written in a clear and (for the most part) concise manner. In general, the authors have used good judgment in deciding where a complete and rigorous presentation is necessary, and where the reader should be referred to the original paper. The articles assume a level of familiarity with solid state theory corresponding to that of an advanced graduate student. Most deficiencies in the reader's general preparation can be remedied by reference to the very well written article on "Methods of the One-Electron Theory." There is a heavy preponderance of theoretical topics in this first volume. The editors plan to correct the imbalance in future volumes.

The article on silicon and germanium is very well organized with a useful bibliography. This article could serve as a model for this type of paper in which experimental data are concisely presented and interpreted. In the article on the theory of order-disorder transitions in alloys, the theoretical material is handled in a very competent and interesting manner. However, the experimental data presented in this article seem neither complete nor sufficiently interpreted. This may only be a reflection of the fact that the experimental aspects of this field have not been attacked with sufficient vigor to enable useful theoretical correlations to be made.

The preponderance of theoretical topics should prove to be a boon to solid state experimentalists. The advances in theory are much more easily digested when presented in the form of well organized review articles than if the scattered original papers must be consulted. This series should serve admirably as a basis for graduate seminar discussions, as well to familiarize graduate students with problems and techniques outside of their thesis topics. This valuable series is auspiciously initiated with the publication of this excellent first volume.

A. E. BERKOWITZ

THERMODYNAMICS, by John Francis Lee and Francis Weston Sears. 543 pages, diagrams, 6 × 9 in. Cambridge, Addison-Wesley Publishing Co., 1955. Price, $7.50.

The principles of thermodynamics are well established in the literature. On the other hand, the presentation of these principles is inherently difficult, that is, the concept of entropy with irreversible systems, so that any new text that clarifies these concepts is welcomed. The authors have definitely contributed to this phase.

"Engineering" thermodynamics usually introduces additional topics such as the coordination of fluid flow (that is, the dynamics of fluid flow) with thermodynamic relations. This book also includes the thermodynamics of the internal combustion engines, refrigeration processes, wherein the application of thermodynamic principles to engineering processes are shown. On this score the book offers an interesting discussion. Thus the mechanical pressure drop equation with friction for flow in a pipe, is correctly given as an integrated momentum equation. The discussion of the Fanno line with friction, and the decrease of enthalpy with increasing entropy for subsonic flow and the increase of enthalpy with increase of entropy with supersonic flow and the critical transition at maximum entropy with the coordination of the flow equations in nozzle design are also well presented.

Coming to the strictly thermodynamic phase of the book, the work gives a more comprehensive presentation on thermodynamic properties and availability relations than is to be found in engineering texts on thermodynamics. The analytical procedures with the thermodynamic coordinates, including the Joule-Thomson (Joule-Kelvin) effect, are well presented. Relative to coordinates, while the first law for steady flow is well illustrated in Fig. 4–7 and corresponding analysis, perhaps, in view of arguments, the "convection" of enthalpy across a section (in place of moving pistons) might have been presented, alternately.

Due to the increasing importance of second law analysis in flow problems, heat transfer equipment, etc., it is felt an improved presentation of both the Second Law, availability and irreversibility relations could be made through the analysis of actual, though simplified, systems. The dual concept of entropy, as a thermodynamic coordinate in heat transfer, with reversible systems, and its growth, with irreversible systems, and as a yardstick for comparing irreversible systems with reversible systems, could be improved in the over-all presentation. These matters could have been well illustrated in the authors' fluid flow analysis, where a property equation with thermodynamic relations, the dynamic or momentum equation and the over-all energy equations including both, along with the concept of growth of entropy, might preferably have replaced the more complicated discussion of Fanno's equation and shock waves.

This book is a considerable advance in an excellent, matured discussion of engineering thermodynamics and should prove adequate for an engineer's background in thermodynamics.

RUPEN EKSERGIAN

MEN, ROCKETS AND SPACE RATS, by Lloyd Mallan. 337 pages, illustrations, 5¾ × 8¼ in. New York, Julian Messner, 1955. Price, $5.95.

With the launching of the instrumented satellites in 1957–1958 as part of the I.G.Y. program man will have taken his first step into space. From these hypersonic man-made moons will come significant data from a complex of extraordinarily ingenious instruments, but the data will be incomplete. The only way in which all the desired information will be acquired is to send up the most intricate, ingenious and versatile of all recorders and that is man himself.

What are the problems involved in sending a man, with his illimitable complexity, into space—even for a short period? The answers to this vital question are given in a 330 page book: *Men, Rockets and Space Rats* by the highly articulate Lloyd Mallan.

The author has traveled 18,000 miles to visit every major installation in the country where researches are mobilized in efforts to crack the space barrier. Among scientists who have lived with the problem of man-in-space there is the unanimous feeling that the weakest link in the space travel chain is the human body. In all these research centers tiny pieces of the jig-saw puzzle, called space physiology, are being carefully studied to determine whether the human body can survive the out-of-this-world condition that will be encountered.

The project the author has undertaken is to arrange these pieces of the jig-saw in their proper places so that an over-all picture can be presented. To do this the author has assembled a wealth of material that in many instances has never been in print. He writes not only of places but of men, both scientists and technicians and this makes the book a veritable "Who's Who" of space flight research personnel. He writes not only of achievements but of hardware—the rockets, rocket planes, centrifuges, decompression chambers, Moby Dick balloons and the acceleration sleds.

He writes of bizarre experiments on men and creatures that to the uninitiated appear callous. But in the unfolding of the experiments the pattern is revealed as a powerful attack on fundamentals which yield, at times, startling results. The author's section on the researches being performed at the Wright Air Development Center is a classic.

All of this is skillfully woven by the author into a superb, stimulating narrative that answers the provocative questions posed today. It is the last word on this new science presented by a writer with a rare competence.

I. M. LEVITT

STATIC AND DYNAMIC ELECTRON OPTICS, by P. A. Sturrock. 240 pages, diagrams, 5½ × 8½ in. New York, Cambridge University Press, 1955. Price, $5.50.

This book is a systematic presentation of the more recent advances in the theoretical techniques of studying the optics of accelerated particles. This subject falls into two natural divisions; namely, Static Electron Optics and Dynamic Electron Optics.

The first division treats problems characterized by steady motion. The bases of calculations for such systems are the Variational Equation and Maxwell's Equations. From these equations may be determined the focussing characteristics of ideal electrostatic and magnetic lenses. An elaboration of the general mechanics of the Hamiltonian theory which evolves from the variational equation is outlined for the analysis of the optical properties of an arbitrary system. Practical examples which are presented as being adaptable to these methods are electron lenses and certain types of deflecting fields which are used in beta-ray spectrometers and mass spectrometers.

The solutions to the variational equation may be represented by characteristic functions, which functions are used to evaluate aberrations and to determine solutions to other more subtle problems such as constructional errors.

This book will be of special interest to those workers concerned with the focussing of charged particles.

ROBERT SAMUEL SMITH

ULTRASONIC ENGINEERING, by A. E. Crawford. 344 pages, diagrams, illustrations, 5½ × 8½ in. New York, Academic Press, Inc., 1955. Price, $8.00.

Ultrasonic Engineering is another addition to the growing list of books which empha-

size the practical aspects of the effects and uses of high intensity vibratory waves. The book reviews and correlates masses of data already published elsewhere, and presents these primarily for the engineer, not the physicist. Beyond giving the reader a hasty background of the applicable basic theory, the author conducts a rather exhaustive survey of British and American exploitations of ultrasonics, especially in industrial processing.

The frequency range of interest in ultrasonic engineering is from about 16 kc/s, just above the normal frequency limit for most persons, to about 500 mc/s, the limit set by available equipment to generate sound waves of appreciate energy. Ultrasonic waves in a liquid medium give rise to the mechanism of cavitation, which is an effect primary to almost all useful applications of ultrasonic energy. This phenomenon is introduced early in the book and its effects are discussed in the chapters on applications.

Four types of generators are available to produce intense ultrasonic waves. These are: piezoelectric or crystal transducers; magneto-striction devices; jet generators; and electromagnetic transducers. For high power, no crystal has surpassed quartz as a generating element, although barium titanate is useful for medium power work. Of the magneto-strictive devices, nickel or nickel alloys exhibit the Joule effect to the greatest degree (change in length of rod along the axis of the change in magnetic field). The design of magneto-strictive rods, circuits, and their losses are considered. For many industrial applications, the simplicity of jet generators employing high velocity liquid or gas streams cannot be overlooked. These make use of resonant cavity or wedge whistles, pulsed liquid jets, or sirens. These are used extensively in the pharmaceutical and food processing industries to produce emulsions, creams, salad dressings, sauces, etc., by either batch or continuous methods.

Seven chapters treat in detail applications in the fields of: precipitation and agglomeration of particles in liquid and gaseous media; emulsification and dispersion; chemical processes such as effects on polmerization and whiskey ageing; metallurgy; coating of metals, especially soldering; biological and medical; and miscellaneous commercial instruments, from laboratory measuring devices to detectors of fish shoals and burglars. Many new applications will arise as more efficient generators are developed.

S. CHARP

ATOMIC PHYSICS, by Gaylord P. Harnwell and William E. Stephens. 401 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1955. Price, $8.00.

As physics advances and expands, it is normal for subjects which were once on its frontiers and topics for advanced seminars to move downward through lower level graduate courses, finally to appear, somewhat simplified, in undergraduate curricula. Sometimes such a regression results in neglect of the texts in the particular field. Older, once advanced texts are allowed to go without revision; new texts are rare, or watered down. Of recent years, atomic physics has suffered such a neglect; the teacher's choice of texts for a dynamic course on the graduate level has been very narrow.

The appearance of a new volume, even if it should prove mediocre, would thus be welcome as providing a wider choice, and perhaps fresh material. But Harnwell and Stephens' book is far better than mediocre, and thus is doubly welcome.

The book is somewhat different from its predecessors in its organization and emphasis. It is not built around either the historic or the experimental approach, though neither point of view is neglected. Rather, it strives to unify the concepts of modern physics, demonstrating the fundamental role of energy and momenta in all of physics. It should disabuse the student of his notion, fostered by much of the undergraduate curricula, that physics is an assemblage of neatly partitioned fields, only loosely interacting, one with another.

Chapter 1 reviews the classical foundations, beginning with a brief but welcome discussion of the role of theory in physics, carrying through relativistic mechanics and closing with the relativistic formulation of electrodynamics. Chapter 2 introduces the idea of discreteness in both matter and radiation. The wave function arises naturally

as an outcome of the description of the wave-like properties of particles. A discussion of angular momenta and magnetic moments concludes the chapter.

Chapter 3 is devoted to Atomic Structure, starting with spectroscopic evidence and continuing with a thorough discussion of the scattering of alpha particles. Following a discussion of the experimental methods for determining energy levels, the Schroedinger Equation is introduced and applied to the hydrogen atom. The treatment here is very complete, carefully demonstrating the properties of the angular momentum operators and introducing the commutator operators. A section on the quantum theory of radiation follows, including time-dependent perturbation theory. Emission and absorption, line breadth, the photoelectric effect, dispersion, and the Raman effect are discussed in some detail.

Chapter 4 covers spin and polyelectronic atoms, using the Dirac spin matrices, and applying the concepts to the explanation of atomic spectra. Molecular structure is treated in Chapter 5, which discusses X-ray scattering, molecular bonds, energy levels, and molecular spectroscopy. Chapter 6 covers classical and quantum statistics with applications to thermodynamics, thermal noise in resistors, black body radiation and the Thomas-Fermi atom.

The last chapter, Chapter 7, is a comprehensive introduction of the properties of matter in all three states. For gases, specific heats, viscosity, heat conduction, magnetic and electric susceptibilities, and the van der Waals equation are covered. The section on liquids is shorter, limited to the transition state and the properties of liquid helium. A more extensive section on the solid state follows, including crystal properties, Einstein and Debye specific heats, electron band theory, electrical conduction, magnetism, and the thermionic and photoelectric effects.

It is a solid text, which will make heavy demands on the student, and which will require some expansion and elaboration by the teacher. But the results should be worth the effort.

A. C. BYERS

THE CHEMISTRY OF SYNTHETIC DYES AND PIGMENTS, edited by H. A. Lubs. ACS Monograph No. 127. 734 pages, 6×9 in. New York, Reinhold Publishing Corp., 1955. Price, $18.50.

The nineteen contributors who assembled the information collected in this monograph have done a real service to all those who need to acquire the latest and most authoritative information on dyes, pigments and intermediates. The contributors who are on the staff of The Organic Chemicals Department of the DuPont Company treat the history, synthesis and uses of each of the classes of dyes, phthalocyanine pigments and dyes in great but not exhaustive detail. The final section of the text by Dr. Donald Graham discusses the ever fascinating subject of the relations between color and chemical constitution of dyes in an entirely satisfactory fashion.

Each chapter ends with voluminous literature references to journals, patent literature and BIOS and FIAT reports. A rough count shows that these reports count for nearly a third of the references. These references alone supply a vast store of hitherto unknown information about German dyes, their uses and manufacture. A section devoted to bibliography lists the BIOS and FIAT reports on dyes and dye intermediates and the major articles on these subjects are arranged in the same order as the sections of the book. There is also an index of dyes and a good subject index.

The reviewer feels that this book has a place in the library of everyone whose work is in the field of dye chemistry or related fields. The text is well written throughout and abundant structural formulas depict the complicated and varied structures of the dyes. The editor and the contributors are to be congratulated for a task well done.

C. Ross BLOOMQUIST

HYDRAULIC OPERATIONS AND CONTROL OF MACHINES, by Ian McNeil. 324 pages, plates, diagrams, 6×9 in. New York, The Ronald Press Co., 1955. Price, $7.50.

The author has presented a general survey of hydrostatic pumps, motors, circuits and auxiliary equipment as applied to presses, extrusion operations, machine tools,

molding and die-casting machines, and mining, agriculture, aeronautical and marine applications. In presenting these topics he describes the elementary theory of operation, the normal range of sizes, operating efficiencies and relative advantages. In many cases interesting historical developments are included.

The broadness of the topic has necessarily limited the discussion of any one subject. For example, his description of servo systems is limited to three pages. In discussing milling machine controls in two pages he described one circuit used to control feed. In this manner he briefly touches upon a large number of topics.

The book is essentially a source of background information on hydraulics at an elemental level. JAMES P. WAGGENER

INTRODUCTION TO ELECTRONIC ANALOGUE COMPUTERS, by C. A. A. Wass. 327 pages, diagrams, 5½ × 7½ in. New York, McGraw-Hill Book Co., Inc., 1955. Price, $6.50.

Introduction to Electronic Analogue Computers is an up-to-date professional report on the status of analogue computers—the philosophy of their design, the design and operation of major components, "tricks of the trade," and other information useful to engineers in the computing field. This book stands almost alone in the clarity of presentation of its material, especially when the author points his attention to the brief description of the theory of operation of elements and their limitations. For the greater part of the book, the author is concerned with simulators, which are: unity time scale machines (operating in real time); d-c. voltage machines (variables of the dynamic problem are represented as corresponding voltages); and explicit machines (each simple mathematical operation is performed by one element of the simulator).

In connection with the solving of sets of differential equations, the author discusses the advantages of using the simulator technique, which allows extension to cases where constant coefficients and nonlinear equations are required to describe the real dynamical system. To illustrate the utility of the simulator approach, the author sets up the computing systems to study such problems as: road vehicle suspension; motion of an airplane; external ballistics; and naval gunnery.

Direct current amplifiers are treated extensively. Detailed operating and design principles are discussed with much emphasis given to the problems of the effects of grid current in the first tube; drift correction, especially by synchronous choppers; input and output impedance characteristics; imperfect circuit functions; and the output limitation of maximum available voltage.

Even though variants of the d-c. amplifier are basic to simulators, nonlinear computing elements are required for many problems. Treated in sufficient detail are: general circuit design for a variable mark/space multiplier, accurate to 1 per cent; electromechanical multipliers; dividers and square root devices; function generators (including the CRT curve follower invented in 1946); and biased diode devices, backlash, limiter, and dead-space simulators.

The principles of setting up and using an electronic analogue computer-simulator are treated in one chapter. Another treats of classes of simulators whose characteristics are slow and fast time scale computers and simulators for networks, all convenient devices for special applications.

This reviewer is of the opinion that there is no comparable material in print, collected together, as has been done in this book. It is highly recommended to all engineers who have an active interest in electronic analogue computers, especially from the design point of view. S. CHARP

MICROSCOPY OF CERAMICS AND CEMENTS, by Herbert Insley and Van Derck Fréchette. 286 pages, illustrations, 6 × 9 in. New York, Academic Press, Inc., 1955. Price, $7.50.

The development of a competent microscope technique cannot be achieved in a library reading room. Only painstaking labors with the microscope can store up the necessary experience for making reliable interpretations. However, this book considerably eases the task of familiarizing oneself with those techniques of microscopy which are applicable to the study of ceramics and cements. The first part of the book briefly describes the functions of the various

kinds of microscopes of interest (polarizing, stereoscopic, metallographic, electron) but the author rightly expects the reader to go elsewhere for detailed studies of these instruments. The general principles of optical mineralogy are then presented in greater detail, although here again it might be well for some readers to refer to the references for some fundamental background.

The remainder of the book is presentation of the technology which is peculiar to the various classifications of ceramics and cements. These classifications include whitewares, refractories, glass, cement, porcelain enamels, clay, etc. The technology consists of preparation for microscopic examination, microstructure examination, and a description of phenomena peculiar to the material (such as the bubble structure of porcelain enamels). ROBERT S. SMITH

COOLING ELECTRIC MACHINES AND CABLES, by Theodore de Koning. 233 pages, diagrams, 6 × 9 in. s'Hertogenbosch (Holland), Zuid-Nederlandsche Drukkerij N.V., 1955. Price: $5.00 (blue cover for individuals and libraries; red cover (for electric manufacturers), determined by author.

This volume is a collection of eight technical papers. It was published at the author's expense in order to introduce his novel ideas on the cooling and the construction of electrical machinery and transmission cables to the engineering fraternity. Two of the papers were previously published in *Electrical Engineering*, the monthly journal of the American Institute of Electrical Engineers. One of these two papers was also printed at about the same time in this JOURNAL.

Mr. de Koning's proposals will seem radical but fascinating to most engineers in the electric power industry. In particular, his suggestion of cooling electrical machines by the vaporization of water, while very intriguing, raises the specter of many fundamental problems in maintaining insulation levels and avoiding corrosion. The author is to be commended for the stimulating and provocative proposals which may require ten or twenty years for practical evaluation.

Since the author published the volume at his own expense, he proposes a novel method of distribution. While individuals may purchase the book at a reasonable price, $5.00 a copy, manufacturers of electrical machinery or cables are required to pay according to the size of the company. The author does not explain how he expects to enforce this rule. BRUCE B. YOUNG

BOOK NOTES

PRINCIPLES OF CHEMICAL ENGINEERING THERMODYNAMICS, by Ernest D. Wilson and Harold C. Ries. 376 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $7.50.

This new text for third year students provides an elementary course in thermodynamics specifically designed for chemical engineering undergraduates. The book is based on 18 years of combined teaching experience of the authors. They have simplified the usual thermodynamics material and have stressed, through problems, the application of thermodynamics to industrial problems. Fourteen chapters cover, in addition to basic thermodynamics topics (such as the first and second laws, the states and properties of matter, etc.), specialized subjects for chemical engineers, including thermochemical effects, phase equilibria, chemical equilibrium, as well as separate chapters on steam power plants and refrigeration. Four appendices contain tables of thermodynamics data, estimation of $\Delta F°_f$ from solubility data, calculation of frictional losses, and a list of problems keyed to each chapter.

PHYSICAL ORGANIC CHEMISTRY by Jack Hine. 497 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $9.00.

This new text on physical organic chemistry was written for industrial chemists and advanced students. It emphasizes the mechanisms of organic reactions and the effect of structure on reactivity. Although about half

the book is devoted to polar reactions, free-radical reactions and four-centered reactions are also treated. Background material—structural theory, acids and bases, and kinetics—is covered in the four chapters of Part I; Part II deals with Polar Reactions in thirteen chapters; Part III consists of six chapters on Free-Radical Reactions; and Part IV contains one chapter on Four-Centered-Type Reactions.

NUCLEAR RADIATION DETECTORS, by J. Sharpe. 184 pages, diagrams, 4 × 6½ in. New York, John Wiley & Sons, Inc.; London, Methuen & Co., Ltd.; 1955. Price, $2.50.

One of the useful Methuen Monographs on Physical Subjects, this little book sets forth the basic information on nuclear radiation detectors. Its six chapters cover a general Introduction, The Interaction of Nuclear Radiation with Matter, Detection Media, The Efficiency of Detectors, Secondary Emission and Scintillation Counters, and Ionization Devices. The monograph should prove useful to those interested in the design and use of detectors for nuclear radiation. Detailed discussion is, of course, excluded by the nature of the work, but each chapter contains a list of references for further study.

BASIC STATISTICAL CONCEPTS, by Joe Kennedy Adams. 304 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1955. Price, $5.50.

Aimed at those students who have had little mathematical training, but who need to know statistics, this book points out the relation between mathematical theorems and applications. Its purpose is to develop basic statistical concepts and to explain the language used in mathematical statistics. Many theorems are presented without proof; proofs of some can be found in the appendix. Among the subjects covered are sampling, statistical inference, distributions (binomial, Poisson, discrete, normal, etc.), and a final chapter, which contains almost no examples, deals with nonparametric statistics.

TRAFFIC ENGINEERING, by Theodore M. Matson, Wilbur S. Smith and Frederick W. Hurd. 647 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1955. Price, $12.50.

Although traffic engineering is only about 20 years old, its problems are so acute and affect so much of our population that this book will be welcomed by all traffic engineers, city planners and State Highway Officials. Here, in one book, can be found authoritative discussions of all the traffic problems we face daily, with suggestions for alleviating the congestion on a scientific basis. The second chapter, on The Road User, is an interesting study of the human behind the wheel—what makes him a good or poor driver. Other subjects covered include parking, intersections, accidents, controls for driver and for vehicle, traffic-control signals, road lighting. Ten chapters discuss the design of traffic-control systems, and five final chapters offer ideas for administration and planning in traffic engineering. The book is practical, well-illustrated, and certainly it is timely and important.

DISTILLATION IN PRACTICE, edited by Charles H. Nielsen. 133 pages, 5 × 7¼ in. New York, Reinhold Publishing Corp., 1956. Price, $3.00.

FLUID FLOW IN PRACTICE, edited by J. R. Caddell. 119 pages, illustrations, 5 × 7¼ in. New York, Reinhold Publishing Corp., 1956. Price, $3.00.

These are the second and third in a series of "Experience in Industry" books which are designed to give young engineers the benefit of older engineers' experience. They are based on symposia sponsored by the Philadelphia-Wilmington Section of the American Institute of Chemical Engineers and the University of Pennsylvania, in May of 1954 and 1955, respectively.

Distillation in Practice covers the most recent developments in distilling techniques, column design, techniques of petroleum fractionation, instrumentation, operation of distillation equipment and commercial aspects of vacuum distillation. The six chapters are all written by authorities from six different industrial firms.

Fluid Flow in Practice is limited to the flow of liquids, including slurries, in pipe-

lines and pumps, and to simple applications for gases. Its seven chapters cover a review of the basic relationships, piping systems, valves, pumps, flow meters, fluid flow equipment and research.

Young engineers in the process industries will find both these books of value in bridging the gap between college training and work in industry.

VECTOR ANALYSIS, by Homer E. Newell, Jr. 216 pages, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1955. Price, $5.50.

This text emphasizes the applications of vector analysis for the engineer and the physicist. Part I covers the theory of scalars and vectors, the differentiation of vectors, divergence and curl, coordinate systems (both rectangular and curvilinear), and potential theory. It includes both vector algebra and vector calculus, and contains many exercises which must be solved by the student if he hopes to become skillful in the use of vectors. Part II is devoted to the applications of vector analysis in such fields as forces, velocities, motions in space, and electromagnetism. The book is so arranged that Part II may be used as illustrative material for Part I, in a one semester, 3-hour course.

ELECTRICAL CIRCUITS, by Charles S. Siskind. 516 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $6.75.

This basic text is divided into two parts, one for d-c. systems, the other for a-c. systems. Profusely illustrated with basic circuit diagrams, the book provides the undergraduate student with the fundamentals of electric and magnetic circuits for machinery, power distribution systems, electronic circuits, control equipment, etc. Each chapter is followed by a section of questions and a set of problems. One feature is an appendix of mathematical derivations of fundamental equations. This work supplements the author's previously published "Electrical Machines"; together, the two volumes are a valuable reference work.

MODERN PHYSICS, A TEXTBOOK FOR ENGINEERS, by Robert L. Sproull. 491 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1956. Price, $7.75.

"Modern" physics—of electrons, atoms and nuclei—has become a necessary part of any engineer's education, no matter what field he is in. The aim of this well-organized book is to give engineers the fundamentals of modern physics and its applications in industry. Divided into 13 chapters and 6 appendices, the book covers, among other subjects, particles (both theory and experiment), quantum mechanics, atomic structure, energy bands, the electrical and magnetic properties of solids, semiconductors, physical electronics and applied nuclear physics. Each chapter has a summary, a bibliography, and a set of from 10 to 41 well chosen problems (no answers are given). Engineers, whether undergraduates or practicing their profession, will find this a useful book to own.

PUBLICATIONS RECEIVED

MATHEMATICAL THEORY OF ELASTICITY, by I. S. Sokolnikoff. Second edition, 476 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $9.50.

PRINCIPLES OF ORGANIC CHEMISTRY, by James English, Jr. and Harold G. Cassidy. Second edition, 469 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $6.50.

MATHEMATICS OF FINANCE, by Paul M. Hummel and Charles L. Seebeck, Jr. Second edition, 372 pages, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $4.75.

NUMERICAL MATHEMATICAL ANALYSIS, by J. B. Scarborough. Third edition, 554 pages, 6 × 9 in. Baltimore, The Johns Hopkins Press, 1955. Price, $6.00.

STEELS FOR THE USER, by R. T. Rolfe. Third edition, 399 pages, illustrations, 5½ × 8½ in. New York, Philosophical Library, 1956. Price, $10.00.

THE PRESERVATION OF NATURAL HISTORY SPECIMENS, edited by Reginald Wagstaff and J. Havelock Fidler. New York, Philosophical Library, 1955. Price, $10.00.

SCIENCE AND MODERN LIFE, by Sir John E. Russell. 101 pages, 5 × 7¼ in. New York, Philosophical Library, 1955. Price, $2.75.

KINETIC THEORY OF LIQUIDS, by J. Frenkel. Unabridged and unaltered republication of the first English translation, 488 pages, 5½ × 8 in. New York, Dover Publications, Inc., 1955. Price: $3.95 (cloth); $1.95 (paper).

PHILOSOPHICAL WRITINGS OF PEIRCE, selected and edited with an introduction by Justus Buchler. Unabridged and unaltered republication of the book "The Philosophy of Peirce: Selected Writings," 386 pages, 5½ × 8 in. New York, Dover Publications, Inc., 1955. Price: $4.50 (cloth); $1.95 (paper).

GEOMETRY OF FOUR DIMENSIONS, by Henry Parker Manning. Unabridged and unaltered republication of the first edition, 348 pages, 5½ × 8 in. New York, Dover Publications, Inc., 1955. Price: $3.95 (cloth); $1.95 (paper).

THE ANALYTICAL THEORY OF HEAT, by Joseph Fourier. Unabridged republication of the English translation of 1878, 466 pages, 5½ × 8 in. New York, Dover Publications, Inc., 1955. Price, $1.95 (paper).

HIGH VACUUM TECHNIQUE, by J. Yarwood. Third edition, 208 pages, illustrations, 5½ × 8½ in. New York, John Wiley & Sons, Inc., 1955. Price, $5.50.

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CURRENT TOPICS

New Transistor with Unsurpassed High Frequency Performance.—A major achievement that opens up new possibilities in electronics has been reported by Bell Telephone Laboratories. Two articles in the January issue of the *Bell System Technical Journal* describe a major breakthrough in transistor technology—new fabricating techniques for an entirely new kind of transistor. These techniques may have far-reaching effects on the use and manufacture of the tiny amplifier, especially for electronic applications in the telephone industry, but in other fields too. The way is now open for the transistor to replace the vacuum tube in many telephone and television jobs.

The new device's performance at very high frequencies surpasses that of any other transistor. Transistor performance has been measured in two principal ways. One criterion has been the number of oscillations or self-generated electrical pulses per second. A second criterion, considered by Bell scientists to be more significant, is the number of voice or communication channels that can be amplified. In this latter category, they report that the new transistor is unequaled.

Experimental units of the device made at Bell's Murray Hill, N. J. laboratories have amplified by 100 to 1 currents across a 20,000,000 cycle wide band. Either the amplification or the number of communication channels can be made three times that of any other transistor. The number of communication channels or pathways varies with the degree of amplification. For example, if the frequency band were cut in half, thus halving the number of channels, there would be twice as much power amplification.

Key to the new fabricating techniques is the development of controls over microscopic chemical layers. The heart of the new transistor is a layer 50 millionths of an inch thick.

The new techniques involve the adaptation of the chemical process of "diffusion" used in treating silicon for the Bell Solar Battery, first device to convert sunlight into substantial amounts of electricity. Diffusion is a process by which minute amounts of impurities are introduced in controlled amounts into a material.

In making the new transistor, an impurity must be introduced only once into the growing crystal. The fully grown crystal receives two other doses of impurity in the easily controlled diffusion process, which is more precise than the older "double-doping" and alloy processes. Both of these processes require control, at very high temperatures, of the boundary between liquid and solid germanium or silicon, in order to determine the dimensions of the base layer. This is very difficult compared with controlling diffusion, in which the desired foreign atoms move through the solid crystal to the required depths.

The transistor consists of a three-layer chemical "sandwich." The center layer is known as the "base." The other two are the emitter and collector layers. The narrower the base layer can be made the higher the frequency at which it will operate. Diffusion provides a high degree of control of such microscopic dimensions.

In considering the practical application of transistors, engineers are concerned with what they call the "frequency cut off." This is the frequency point up to which there is straight, full amplification of a signal and after which the signal is amplified with steadily diminishing strength.

Currently available transistors have a frequency cut off of 1–10 megacycles, and several recently anounced transistors have had a frequency cut off between 100 and 200 megacycles (100–200 million cycles). Thus far, the new transistors have been found to reach a cut off between 500 and 600 megacycles. Even though the effective operation of many military electronic devices is in a much lower megacycle range, the higher frequency cut off would provide a greater number of communication channels or more amplification than previously possible.

Because of its very high frequency characteristics the new transistor appears to be ideally suited for application in guided missiles and electronic "brains" for military and computer uses.

Tradic, an electronic computer designed by Bell Laboratories for military applications, already uses a transistor of a different type, and transistors are being used in other military equipment. While these transistors are well suited for the jobs they do, they cannot be used for a large number of other applications. The new transistor opens up many new possibilities for military technology.

"Small" Brain Assists "Giant" Brain.—Something new in electronic computing—using a "small brain" to assist a "giant" in solving complex problems quickly—got underway recently at the United Aircraft Computation Laboratory, East Hartford, Conn. United Aircraft engineers began using a Burroughs Corporation E101 electronic computer, which is the size of an office desk, to pilot the calcutions of a giant robot, which fills a large room, in designing advanced jet engines.

Stuart L. Crossman, head of the Computation Laboratory, explained that the Burroughs "small brain" pilot operations are part of a process known in the electronic computer field as "debugging the program"—checking the accuracy of instructions fed into the "giant brain."

Mr. Crossman estimated that the use of Burroughs' E101 will save hard-to-find computer personnel "from several hours to several weeks" in solving the hundreds of complicated mathematical problems involved in the design of aircraft engines and other aircraft components.

One of the E101's major attractions is that it does not require highly trained operator personnel. Because of that, Mr. Crossman said, the small computer is "well suited for departments where design and other engineering problems originate but whose personnel have no computer training. The E101 is about as easy as anything can be and still be an electronic computer. It can fit in a small office and anyone can run it.

Engineering Applied to Human Beings.—The principles which engineers have used to build great bridges are now being applied to the human mechanism in solving orthopedic problems. Plans to study the structural and mechanical functions of the bones, joints, and muscles—and the problem of braces and other mechanical aids for the body—have been announced by the Massachusetts Institute of Technology in cooperation with the Massachusetts General Hospital. A $10,000 grant to M.I.T. by the Donner Foundation, Inc., of Philadelphia will finance a two-

year research program on engineering in orthopedics.

Commenting on the new program, Dr. Robert J. Hansen, associate professor of structural engineering at M.I.T., pointed out that "engineering problems in orthopedics are many, varied, and complex." This fact has been recognized by the medical profession and has led to research programs including studies of the function, design, and operation of artificial limbs, braces, and other mechanical aids used both externally and internally; the adaptation of various materials for use within the body; and investigations of the mechanical function of the skeletal and muscular systems.

"The Department of Civil Engineering at M.I.T. has cooperated with the Massachusetts General Hospital in the past on a number of orthopedic problems. Among these has been a study of the function of the lumbar spine, the action and design of leg braces, and the function of back braces in immobilizing the lower spine."

The first assignment under the new program, Dr. Hansen says, will be to outline the areas of orthopedics to which engineering may make the greatest contribution. Recent advances, he says, indicate that mechanics and materials applied with sound engineering principles will speed the recovery and increase the mobility of many patients.

What, for instance, is the cause of low back pain which is so common in middle age? Can an engineering approach assist in developing better methods of treatment for this common and perplexing problem?

In general the studies contemplated under the new program at M.I.T. will deal with the human mechanism from the standpoint of forces and movements. When these are better understood, Dr. Hansen believes, it may well be possible to apply engineering principles to the construction of improved artificial joints, to the bracing of weakened parts of the skeleton, and to the restoration of function to defective limbs.

Plane Carried Rockets for Research.—The Office of Naval Research's program to gather information on the upper atmosphere has been extended to include plane-launched rockets. Recently, the Naval Air Station, Chincoteague, Virginia, was the scene of tests in which rockets were launched vertically from a Navy "Banshee" jet fighter at an altitude of 30,000 ft. The rocket then launched proceeded to an altitude of 90,000 ft.

The new rocket-aircraft method, called "Rockair," will provide a practical and inexpensive tool for research observations at high altitudes. During the feasibility tests, only cosmic ray studies were made. There are, however, many possible applications of the new plane-rocket method. Useful information can be obtained, for example, on ambient air temperatures, air densities, ozone concentrations, and high altitude winds.

New Supersonic Wind Tunnel.—The University of California has a new supersonic wind tunnel at its engineering field station located at Richmond, Calif. This wind tunnel—200 feet long, 3 feet in diameter, and capable of testing speeds up to Mach 6—is the largest of its kind in the world and represents the result of combined planning and financial support by the Office of Naval Research, the National Advisory Committee for Aeronautics, the Office of Scientific Research, and the Air Research and Development Command of the United States Air Force.

Prefreezing Wax Dip for Juicier Meat.—In the first commercial test of its kind, processors are trying to improve the "freezability" of meat and poultry by predipping cuts in wax, an oil product. Food experts say that the wax coating seals in juices, thereby preventing the meat from drying out during the freezing process. When the coating is removed days or months later, the meat is still red and juicy. Wax will be no problem to the housewife, since it can be peeled off easily without sticking to the meat. This wax, an oil-based microcrystalline variety specially modified for food processing, is one of the many special petroleum products that are being developed constantly to create better living for American consumers.

Ultrasonics Speeds Production of Insulation.—The General Electric Company has successfully used ultrasonics—sound waves inaudible to the human ear—to speed up the process of making electrical insulation.

George E. Henry of the company's General Engineering Laboratory in Schenectady said tests showed ultrasonic treatment of a typical material to be 10 times faster than treatment by normal methods.

The new technique, he said, may lead to significant cost reductions in treating materials used in electrical insulation. But he cautioned that further developments must be made before the process is economically feasible on a broad scale.

G.E. has pioneered in the development of ultrasonics as a tool of industry and engineering. The powerful sound waves have been used for cleaning minute or inaccessible parts, for bonding of materials and for other jobs.

Henry made the disclosure of ultrasonic impregnation in an address before a joint meeting of the Binghamton chapters of the American Institute of Electrical Engineers and the American Society of Mechanical Engineers. He spoke on the general subject of ultrasonics in industry.

Under the new impregnation technique, Henry said, semi-porous materials such as paper and cloth are treated speedily for insulation purposes. This requires replacing the air and water content of the material with an insulating liquid such as varnish, resin or asphalt.

Sound waves from an ultrasonic generator are beamed through the liquid. The material to be treated is dipped into the liquid in the path of the sound wave.

The ultrasonic beam agitates the liquid with a to-and-fro motion that changes the direction of the liquid's particles a million times a second, Henry said.

The conventional method of impregnating paper or cloth is to subject it to a pressure-vacuum cycle in the presence of the liquid impregnant. This method calls for applying pressure and then vacuum alternately, and usually is time-consuming.

"The ultrasonic process sometimes is applied to the material in bulk quantity and on other occasions after the material has been assembled into an electrical unit, depending on which is the more practical," Henry said.

Impregnation of coil windings in a small transformer is an example of treatment after assembly into an electrical unit, he added.

The process also can be used in treating materials for other uses, such as laminar plastics. Bulk treatment is more effective in the preparation of plastics such as those used on kitchen counters, he said.

Impregnation, he added, "does more than just improve the material as an insulator. It also offers an advantage

in mechanical strength and in isolating a possible source of noise from the ears of a sensitive listener."

But further improvements are necessary to make ultrasonic impregnation economically practical in the automatic processing of insulation materials, the G–E expert said.

"These improvements," he explained, "will have to come in the equipment rather than in the technique. However, ultrasonics is becoming an increasingly significant factor in the growing trend toward automation."

Crevasse Detector Developed for Operation Deepfreeze.—A Navy Chief using an Army engineer's design and blueprints has constructed a crevasse detector for Operation Deepfreeze which has been found 100 per cent successful. Owen M. Perry, Chief Electronics Technician, United States Navy, obtained the necessary parts and assembled the detector as he traveled aboard an icebreaker enroute to Antarctica.

A crevasse is an open chasm in glacial or land ice usually covered by snow. Parties passing on foot, sled, or tractor in the past have lived in constant fear of falling into these pits.

With the new detector, the snow vehicle driver proceeds along the ice and watches a special meter. Normal meter reading is 20. When the needle dips to 15 he knows the ice is suspicious. Readings between 15 and 10 indicate danger. Ten or below means "bail out." On tests the detector spotted every known crevasse and even some new ones.

Here is how the device works. One metal sled, 20 feet in front of the snow vehicle, or Weasel, holds a 39-in. square detector. Two other metal sleds trail the Weasel at 20-ft. intervals. An electric impulse is released from the after sled, energy passes through the snow under the vehicle, in the air over the vehicle, and completes the cycle at the point of origin. The cycle reverses itself 60 times each second. Thus, normal snow underfoot produces a meter reading of 20. An empty spot in the ice hinders the impulse and a lower reading results.

With the detector mounted 20 ft. in front of the Weasel, the driver has only ten feet stopping area before the vehicle reaches the edge of the crevasse when a low reading is recorded. However, since the Weasel travels only about three miles per hour, ten feet is sufficient stopping space.

Oil Man's Yarn—Highly Colored.—For the first time in the history of man-made fibers, a method has been devised to make acetate yarns colorfast and virtually fade-proof. These fibers, which are derived in part from petroleum, are actually created in color, rather than being dyed later as are conventional fabrics. The technique called "solution dyeing," is accomplished by forcing a colored chemical solution into the bath which hardens the flowing plastic into a continuous thread. Thus laboratories spin materials derived from petroleum into new products to make American homes more comfortable and attractive.

Toll Collection while Vehicles are in Motion.—The steady beam of an electronic eye is a key feature of a new toll collection and audit system developed by the International Business Machine Corporation and installed on the recently completed Ohio Turnpike.

At all interchanges and terminal points along the new expressway, the system both weighs and classifies the vehicles automatically. The system operates while the cars and trucks are in motion—even when following bumper to bumper.

A photoelectric eye system, developed by the General Electric Company, automatically counts the number of axles per vehicle. Weight per axle is determined by a specially designed treadle built into the pavement of the traffic lane.

Vehicles are classified according to individual axle weights, added together.

Engineered by IBM, the new system provides increased revenue protection, greater operating efficiency and improved vehicle classification.

It works like this:

An attendant inserts a fare card into a toll recorder, determines by looking at the passing car or truck how many axles it has, and pushes the appropriate key on the recorder.

Pushing the proper key automatically totals the individual axle weights. It also registers the vehicle's classification on the fare card—along with other necessary information.

When the attendant pushes the key, an "axle group indicator" on top of the toll recorder has already received the correct information as to the number of axles from the General Electric photoelectric system. Should the wrong key be punched, the axle group recorder lights up and tells the attendant the correct number of axles and at the same time locks to prevent improper validation of the fare cards.

The fare card is given to the vehicle operator who presents it at the exit gate, along with his toll.

The system automatically reweighs the vehicle at the exit interchange and compares the weight class punched by the entrance recorder with the class as determined by the exit classifier. The equipment also reads the number of axles and the time punched by the entrance recorder.

All weighing and counting operations are performed while the vehicles are in motion.

Ultraviolet Microscope.—Norden-Ketay's automatic color translating ultraviolet microscope is available for accurate structure determination and indication of absorption spectrum.

The ultraviolet microscope is the newest means for studying not only the geometry, but also the chemical similarities and dissimilarities of objects within specimens of various types. It is useful in investigations in biology, medicine, physics, metallurgy, and many other fields.

New VHF-UHF Antenna.—Under an authorization from the Bureau of Ships, Edward N. Singer and Herschel R. Caler have developed a new broadband antenna and balun for the 88–400 megacycle frequency range.

The antenna was designed for use with field intensity meters which are applied to the study of radio interference, television receiver radiation, transmitter harmonics, and general field intensity surveys.

When a standard resonant dipole antenna is used for measurements of this sort, the dipole antenna requires several adjustments for every individual frequency measured. However, the frequency range of a dipole antenna can be varied by the addition and adjustment of from one to four pairs of antenna elements. The elements of the antenna must then be measured and adjusted to its proper length corresponding to the desired frequency.

In addition, a balun shorting stub, when present, must also be measured and adjusted to its proper length. A "balun" is an electrical-circuitry which couples or matches a balanced radio source to an unbalanced radio frequency load or vice-versa, with a minimum loss of energy; hence the abbreviation "bal-un." The purpose of a balun is to enable a balanced dipole

antenna to be used with a coaxial cable.

While the antenna constructed was for the specific frequency range of 88–400 megacycles, the design has been calculated in general terms so that similar antennas may be constructed for any 4.5:1 radio frequency coverage in the UHF and VHF spectrum.

Complete particulars as to deviation of measurements, construction details, and dimensions were published by Singer and Caler in the December issue of the *Bureau of Ships Journal.*

Histological Freeze Drying Unit. In pathology, anatomy, cytology, and embryology, freeze drying is the best available method for permanently "fixing" tissue characteristics. It overcomes physical distortion and chemical alterations within individual cells and in tissue structure. Cellular and histological details are more accurately preserved and more readily seen. This is due principally to the avoidance of shrinkage. Tissue constituents are immobilized by quick freezing; chemical reactions are stopped essentially instantaneously. Results have shown that sensitive chemical alterations such as denaturation can be avoided by freeze drying. Freezing is accomplished under conditions in which ice crystal formation is of submicroscopic size.

Boston Electronics Division of the Norden-Ketay Corporation offers a histological freeze drying unit for use in cytochemistry and histochemistry (including autoradiography). In addition to providing superior cytology, to avoiding chemical changes during fixation, and to preventing artifacts due to leaching and displacement within cells and tissues, a permanent preparation, with all of the well known advantages of such preparations, is obtained.

The model FD-11 unit is small, compact, semi-automatic, and offers excellent visibility. A simple, sensitive electronic vacuum gauge is included. Drying periods are regulated by size of tissue (20 permanent blocks or more can be prepared in 48 hours). The unit provides simple and efficient infiltration under vacuum, and gives consistently reproducible results.

Ultrasonic Inspection.—The first automatic ultrasonic inspection equipment for inspecting axial flow rotors of jet engines will soon be turned over to the United States Air Force's Air Materiel Command for use in overhaul operations.

Using techniques developed by General Electric engineers of the Evendale Operating Department at the Company's jet engine plant near Cincinnati, O., Electro Circuits, Inc., of Pasadena, designed and built the new equipment. Application of this equipment is expected to produce time and labor savings in the inspection of the many thousands of G-E J47 engines now in service in the Air Force.

First use of the equipment will be in the overhaul operations of Norton Air Force Base at San Bernardino, Calif., according to the Air Force.

Without dismantling the compressor rotor, an inspection can be made in one hour with the new equipment, compared to about eight hours required to perform an ultrasonic inspection manually.

The equipment is designed to inspect the rear wheel lip of all rotor stages for cracks, record the location and signal of the probe at all times throughout the inspection, automatically scan each wheel and inform the machine operator when inspection of wheel is complete, and show visually the position and signal from the probe at all times. The unit is also designed to

detect a $\frac{1}{8}$-in. hole as little as 0.040 in. from a surface.

Ultrasonic frequencies used can be 5, 10, 15, 20, or 25 megacycles, depending on the type of material being inspected. Water is used as a coupling agent and only the section of the rotor being inspected is immersed in the water. A 17-in. cathode ray tube is used to give a visual report of the inspection along with a recorder that can picture the shape of any crack detected.

The equipment operates on 110 volt, 60 cycle, single phase, or 220 volt, 60 cycle, three phase power.

Automatic Dispatching Systems for Electric Utility Companies.—A new automatic dispatching system—the first to perform the routine and repetitive functions of electric utility production departments as well as automatically regulate power generation for fluctuating customer needs—has been announced by General Electric's Instrument Department.

The new dispatching system consists basically of a remote-position repeating selsyn circuit which requires only a single control channel to each generating station. It adjusts total generation to maintain desired average conditions of frequency and tie-line load.

Moreover, the dispatching system allocates total generation between machines automatically in order to realize a minimum fuel input to the system. This is based on a continuous use of incremental cost data for every machine under control. The system production cost (mills/KWH) is compared with this incremental cost data for each machine to specify a value of output. The economic features of this control system comprise an analogue computer which continuously specifies an optimum division of output among all machines.

To assure that total generation is properly adjusted to match the area load, measurements of tie-line power and system frequency are combined in order to detect any deviation from desired conditions. Any deviation is employed by the master area controller to correct the prevailing incremental cost signal to restore the power system to balance.

The dispatching system is composed of four console sections. These include: master area, dispatching, master station, and turbine control console sections.

The master area section contains all controls for determining the mode of operation for the system as a whole.

The dispatching section contains all controls for inserting transmission-loss penalty factors and means for providing signals for the wire line, carrier or microwave channel to each station.

The master station section has all controls for determining the mode of operation of the station as a whole.

The turbine control section has all controls for determining the mode of operation of individual turbines and for inserting the complete incremental cost data for the individual turbine and boiler combination.

Helicopter as Mine Countermeasure Tow Vehicle.—With the advent of the helicopter as a towing vehicle, the battle against the sea mine, that ancient but very effective scourge of the sea lanes, has entered a new phase. The helicopter has demonstrated its ability to tow mine countermeasure devices effectively.

By reason of the fact that the helicopter operates with a high degree of mobility outside of the medium of the mine, the element of danger to per-

sonnel and equipment, which has been ever present in mine countermeasure operations, has been drastically reduced.

The pulling power of helicopters has even amazed those who are most familiar with them. Forty to four hundred ton surface vessels have been towed from standing starts at speeds considerably in excess of that which they can produce under their own power. The Armament Division of the Bureau of Aeronautics, which was responsible for the concept of the use of helicopters in mine countermeasures, is vigorously expanding its program to broaden the scope of the use of the helicopter in this field.

The advent of the helicopter as a mine countermeasure vehicle constitutes a major contribution to the solution of the Navy's prime problem of keeping the sea lanes open.

Ni-O-Nel Electrode and Filler Wire.—Two new welding products recently developed at the Bayonne Research Laboratory of the International Nickel Company now make it possible to weld Ni-o-nel nickel-iron-chromium alloy. The products are "135" Ni-o-nel electrode and "65" Ni-o-nel filler wire and rod.

The electrode and filler wire rod enable the welder to lay a sound, high-quality bead. Both products give X-ray quality welds and allow operation in all positions. The welds have corrosion resistance and strength equal to that of the base metal itself.

Made in four convenient diameters, $\frac{3}{32}$-in., $\frac{1}{8}$-in., $\frac{5}{32}$-in., and $\frac{3}{16}$-in., the "135" Ni-o-nel electrodes come in 14-in. lengths.

The "65" Ni-o-nel filler wire is produced in diameters of 0.035, 0.045, and 0.062 in., on disposable spools for inert gas welding consumable process. Produced in 36-in. straightened and cut lengths for tungsten-arc process, it is available in 5 diameters: $\frac{3}{16}$-in., $\frac{5}{32}$-in., $\frac{1}{8}$-in., $\frac{3}{32}$-in., and $\frac{1}{16}$-in.

Ni-o-nel, which was released by Inco to the trade in April, 1955, is a high nickel alloy containing approximately 40 per cent nickel, 21 per cent chromium, 3 per cent molybdenum, 1.75 per cent copper, 31 per cent iron and small amounts of manganese, silicon and carbon. The relatively high nickel content, plus molybdenum and copper, makes this alloy considerably more resistant to hot sulphuric, sulphurous and phosphoric acid solutions than any of the common stainless steels. In addition, the chromium content is sufficient so that when fortified with nickel, the alloy is capable of resisting a wide variety of oxidizing chemicals.

Ni-o-nel is used in the sulphuric acid pickling of iron, steel and copper-base alloys; in petroleum refineries, synthetic detergent plants, rayon manufacture, ore processing plants, and the pulp and paper industries.